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Two-loop effective potential for a general renormalizable theory and softly broken supersymmetry

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Abstract

I compute the two-loop effective potential in the Landau gauge for a general renormalizable field theory in four dimensions. Results are presented for the $\overline{\rm MS}$ renormalization scheme based on dimensional regularization, and for the $\overline{\rm DR}$ and $\overline{\rm DR}'$ schemes based on regularization by dimensional reduction. The last of these is appropriate for models with softly broken supersymmetry, such as the Minimal Supersymmetric Standard Model. I find the parameter redefinition which relates the $\overline{\rm DR}$ and $\overline{\rm DR}'$ schemes at two-loop order. I also discuss the renormalization group invariance of the two-loop effective potential, and compute the anomalous dimensions for scalars and the beta function for the vacuum energy at two-loop order in softly broken supersymmetry. Several illustrative examples and consistency checks are included.

Contents

1 Introduction

The Fermilab Tevatron collider and the CERN LHC collider hold the promise of exposing the nature of spontaneous electroweak symmetry breaking. In the Standard Model, this mechanism relies on a non-zero vacuum expectation value (VEV) for a fundamental Higgs scalar field. There are good theoretical and experimental reasons to suspect that this picture is correct, but incomplete, and must be embedded in a larger theory such as supersymmetry [1, 2]. When new experimental discoveries are made, the tasks of telling the difference between different candidate models of electroweak symmetry breaking and constraining the underlying parameters of the successful theory will require high-precision calculational tools at the two-loop level or better.

The effective potential [3]-[5] allows the calculation of the VEVs in the true vacuum state of a theory with spontaneous symmetry breaking. In this formalism, the scalar fields of the theory are each separated into a constant classical background ϕ plus quantum fluctuations. The effective potential $V(\phi)$ is equal to the tree-level potential in the classical background, plus the sum of one-particle-irreducible connected vacuum graphs. These are calculated using the Feynman rules with ϕ-dependent masses and couplings. Thus one may write

$$V = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)} + \ldots, \tag{1.1}$$

where $V^{(n)}$ represents the n-loop correction.† In this paper, I will be concerned with the effective potential in Landau gauge. Although the effective potential itself is gauge-dependent, physical properties following from it, such as its value at stationary points, and the question of whether or not spontaneous symmetry breaking occurs, are gauge invariant [6]. The one-loop contribution $V^{(1)}$ is well-known for a general field theory, and is reviewed in section 3. In ref. [7], Ford, Jack and Jones have calculated $V^{(2)}$ in the special case of the Standard Model using dimensional regularization (DREG) with minimal subtraction or modified minimal subtraction ($\overline{\rm MS}$). Their calculations can be generalized to obtain the corresponding result for any renormalizable field theory, as I will do explicitly in section 4.

However, it is well-known that the DREG regularization method is not convenient for theories based on supersymmetry. This is because in DREG, the vector field only has $4 - 2\epsilon$ components, introducing a spurious non-supersymmetric mismatch with the number of degrees of freedom of the gaugino. Therefore, in DREG the relationships between couplings which should hold in a softly broken supersymmetric theory are violated even at one-loop order. Instead, one can use the dimensional reduction (DRED) method [8], in which loop integrals are still regularized by taking momenta in $4 - 2\epsilon$ dimensions, but all 4 components of each vector field are kept. The extra 2ϵ components of the gauge field in DRED transform like scalars in the adjoint representation of the gauge group, and are known as epsilon

†To save ink, a factor of $1/(16\pi^2)^n$ is always factored out of the n-loop contribution to the loop expansion of the effective potential, β-functions, and anomalous dimensions in this paper.

scalars. The renormalization scheme based on DRED with modified minimal subtraction is known as $\overline{\rm DR}$. It has the virtue of maintaining manifest supersymmetry in theories where supersymmetry is not explicitly broken.

Realistic models of the physics at the TeV scale do involve explicit soft violations of supersymmetry, however. In such models, the $\overline{\rm DR}$ renormalized dimensionless couplings of the theory obey the relations prescribed by unbroken supersymmetry. However, the epsilon scalars in general do not have the same masses or dimensionful couplings as do the ordinary $4-2\epsilon$ vector field. In fact, computation of the renormalization group (RG) equations shows that the running squared masses of the epsilon scalars cannot be consistently set equal to those of the corresponding vector gauge bosons [9]. This makes the $\overline{\rm DR}$ scheme also inconvenient, since the epsilon-scalar masses are unphysical. A better scheme is the $\overline{\rm DR}'$ scheme [10], which differs from $\overline{\rm DR}$ by a parameter redefinition. The $\overline{\rm DR}'$ scheme offers the advantages that the epsilon-scalar masses completely decouple from all RG equations, and also from the equations that relate running renormalized parameters to pole masses and other physical observables.

In this paper, I will present results for the two-loop effective potential in the Landau gauge and in each of the $\overline{\rm MS}$, $\overline{\rm DR}$, and $\overline{\rm DR}'$ renormalization schemes. For models with exact supersymmetry, the last two schemes are the same, while for models with softly broken supersymmetry the $\overline{\rm DR}'$ scheme is by far the most convenient.

The topologies of the one-particle-irreducible connected vacuum graphs at one- and two-loop orders are shown in Figure 1. Because the one-loop graph topology does not involve interaction vertices, $V^{(1)}$ clearly depends only on the field-dependent squared masses m_n^2, where the index n runs over all of the real scalars, two-component fermions, and vector degrees of freedom in the theory. Note that any complex scalar can be written in terms of two real scalars, while four-component Dirac and Majorana fermions can always be written in terms of two-component left-handed Weyl fermions, in a way throughly familiar to disciples of supersymmetry (see refs. [1, 2] for a discussion). In any dimensional-continuation regularization scheme, quadratic divergences are automatically discarded, and one finds for the renormalized effective potential at one-loop order:

$$V^{(1)} = \frac{1}{4}\sum_n (-1)^{2s_n}(2s_n+1)(m_n^2)^2(\overline{\ln} m_n^2 - k_n). \tag{1.2}$$

Here I have adopted the notation

$$\overline{\ln}(x) = \ln(x/Q^2), \tag{1.3}$$

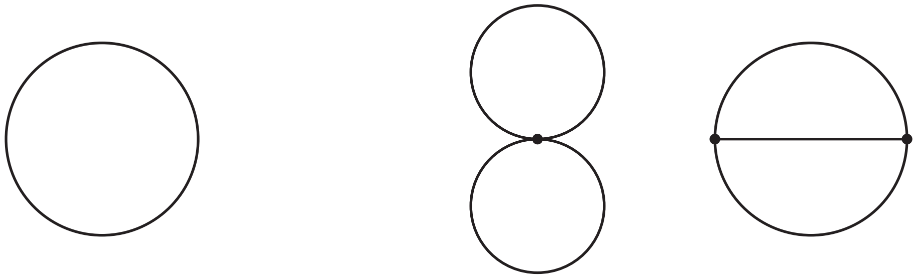

Figure 1: Topologies of one-particle-irreducible connected vacuum Feynman diagrams for the one-loop and two-loop contributions to the effective potential.

where Q is the renormalization scale, and $s_n = 0, 1/2, 1$ for real scalars, two-component fermion, and vector degrees of freedom respectively,[‡] and k_n are constants depending on the details of the renormalization scheme.

From figure 1, it is clear that at two-loop order the result must be of the form

$$V^{(2)} = \sum_{n,p} g^{nnpp} f_{np}(m_n^2, m_p^2) + \sum_{n,p,q} |g^{npq}|^2 f_{npq}(m_n^2, m_p^2, m_q^2), \tag{1.4}$$

where g^{npqr} and g^{npq} are field-dependent four- and three-particle couplings, and $f_{np}(x, y)$ and $f_{npq}(x, y, z)$ are Q-dependent functions obtained by performing the appropriate two-loop integrations. So the task is to identify these objects for each combination of particle types that can contribute.

The rest of this paper is organized as follows. Section 2 describes the field-dependent masses and couplings, lists the relevant Feynman diagrams, and presents necessary conventions. Section 3 reviews the one-loop effective potential, distinguishing between the $\overline{\rm MS}$, $\overline{\rm DR}$, and $\overline{\rm DR}'$ schemes. Sections 4-6 present the results for the two-loop effective potential contribution in each of those schemes. Section 6 also explicitly gives the redefinitions necessary to go from $\overline{\rm DR}$ to $\overline{\rm DR}'$. Section 7 discusses the RG invariance of the effective potential in the $\overline{\rm DR}'$ scheme, and derives some necessary results for the scalar anomalous dimension and vacuum energy beta function in softly broken supersymmetry. Section 8 contains some illustrative examples and consistency checks.

[‡]The contribution of epsilon scalars is discussed in section 3.

2 Conventions and setup

2.1 Field-dependent masses and couplings

Let us write the quantum fields of a general renormalizable field theory as a set of real scalars R'_i, two-component Weyl fermions ψ'_I, and vector fields A'^{μ}_a. Scalar flavor indices are $i, j, k, \ldots$; fermion flavor indices are $I, J, K, \ldots$; and $a, b, c, \ldots$ run over the adjoint representation of the gauge group. Space-time vector indices are written as Greek letters $\mu, \nu, \rho, \ldots$. I use a metric with signature $(-{+}{+}{+})$, and the notations for fermions follow [1, 2]. The primes are used to indicate that these fields are not squared-mass eigenstates. The kinetic part of the lagrangian includes

$$-\mathcal{L} = \frac{1}{2} m^2_{ij} R'_i R'_j + \frac{1}{2}(m^{IJ} \psi'_I \psi'_J + \text{c.c.}) + \frac{1}{2} m^2_{ab} A'^{\mu}_a A'_{\mu b}. \tag{2.1}$$

The symmetric fermion mass matrix m^{IJ} yields a fermion squared-mass matrix

$$m^2_{IJ} = m^*_{IK} m^{KJ}. \tag{2.2}$$

Then m^2_{ij} and m^2_{ab} are real symmetric matrices, and m^2_{IJ} is a Hermitian matrix, and in general they all depend on the classical background scalar fields. In order to calculate the effective potential, the first step is to rotate to squared-mass eigenstate bases R_i, ψ_I, A^{μ}_a. This can be done by using orthogonal matrices $N^{(S)}$, $N^{(V)}$ for the scalar and vector degrees of freedom, and a unitary matrix $N^{(F)}$ for the fermion degrees of freedom. So, the rotations

$$\begin{aligned} R'_i &= N^{(S)}_{ji} R_j, & (2.3)\\ \psi'_I &= N^{(F)*}_{JI} \psi_J, & (2.4)\\ A^{\mu\prime}_a &= N^{(V)}_{ba} A^{\mu}_b, & (2.5) \end{aligned}$$

are chosen such that:

$$\begin{aligned} N^{(S)}_{ik} m^2_{kl} N^{(S)}_{jl} &= \delta_{ij} m^2_i, & (2.6)\\ N^{(F)}_{IK} m^2_{KL} N^{(F)*}_{JL} &= \delta_{IJ} m^2_I, & (2.7)\\ N^{(V)}_{ac} m^2_{cd} N^{(V)}_{bd} &= \delta_{ab} m^2_a. & (2.8) \end{aligned}$$

Here m^2_i, m^2_I, and m^2_a are respectively the scalar, fermion, and vector squared-mass eigenvalues which will appear in the effective potential. It should be noted that in general $N^{(F)}$ diagonalizes the fermion squared-mass matrix m^2_{IJ}, but need not diagonalize the fermion mass matrix m^{IJ}. All that is required is that

$$M^{IJ} = N^{(F)*}_{IK} m^{KL} N^{(F)*}_{JL} \tag{2.9}$$

has a block diagonal form, with non-zero entries only between states with the same squared-mass eigenvalue. Indeed, it is quite often not particularly desirable for $N^{(F)}$ to diagonalize the fermion mass matrix, for example in the case of charged Dirac fermions, with doubly-degenerate eigenvalues for m_I^2, where M^{IJ} is best left off-diagonal in 2×2 blocks. The matrix M^{IJ} and its complex conjugate M^*_{IJ} will appear as mass insertions. In practical applications, the diagonalizations just described are easily performed numerically using a computer, and under favorable circumstances (such as those studied in section 8) they can be done analytically. In either case, the problem amounts to finding the orthonormal eigenvectors of m^2_{ij}, m^2_{IJ}, and m^2_{ab}.

Now the interaction terms in a general renormalizable theory can be written in terms of the squared-mass eigenstate fields as

$$\mathcal{L}_{\rm S} = -\frac{1}{6}\lambda^{ijk} R_i R_j R_k - \frac{1}{24}\lambda^{ijkl} R_i R_j R_k R_l, \tag{2.10}$$

$$\mathcal{L}_{\rm SF} = -\frac{1}{2} y^{IJk} \psi_I \psi_J R_k + \text{c.c.}, \tag{2.11}$$

$$\mathcal{L}_{\rm SV} = -\frac{1}{2} g^{abi} A^a_\mu A^{\mu b} R_i - \frac{1}{4} g^{abij} A^a_\mu A^{\mu b} R_i R_j - g^{aij} A^a_\mu R_i \partial^\mu R_j, \tag{2.12}$$

$$\mathcal{L}_{\rm FV} = g^{aJ}_I A^a_\mu \psi^{\dagger I} \overline{\sigma}^\mu \psi_J, \tag{2.13}$$

$$\mathcal{L}_{\rm gauge} = g^{abc} A^a_\mu A^b_\nu \partial^\mu A^{\nu c} - \frac{1}{4} g^{abe} g^{cde} A^{\mu a} A^{\nu b} A^c_\mu A^d_\nu + g^{abc} A^a_\mu \omega^b \partial^\mu \overline{\omega}^c, \tag{2.14}$$

where ω^a and $\overline{\omega}^a$ are massless (in Landau gauge) ghost fields. This defines the field-dependent couplings to be used in the two-loop effective potential calculation. The scalar interaction couplings λ^{ijk} and λ^{ijkl} are each completely symmetric under interchange of indices, and real. The Yukawa couplings y^{IJk} are symmetric under interchange of the fermion flavor indices I, J. The remaining couplings all have their origins in gauge interactions. The vector-scalar-scalar coupling g^{aij} is antisymmetric under interchange of i, j. The pure gauge interaction g^{abc} is completely antisymmetric; it is determined by the original gauge coupling g, the antisymmetric structure constants f^{abc} of the gauge group, and $N^{(V)}$, according to

$$g^{abc} = g f^{efg} N^{(V)}_{ae} N^{(V)}_{bf} N^{(V)}_{cg}. \tag{2.15}$$

Similarly, if the fermions ψ'_I transform under the gauge group with representation matrices $(T^a)^J_I$, then the vector-fermion-fermion couplings are

$$g^{aJ}_I = g (T^b)^K_L N^{(F)*}_{JK} N^{(F)}_{IL} N^{(V)}_{ab}. \tag{2.16}$$

Note that even the dimensionless couplings generically depend on the classical scalar background fields ϕ, through their dependence on the rotation matrices $N^{(S)}$, $N^{(F)}$, and $N^{(V)}$.

2.2 The Feynman diagrams

The two-loop effective potential is to be evaluated by computing the one-particle-irreducible connected vacuum Feynman diagrams appearing in figure 2. The masses and couplings of fields appearing in these diagrams are as indicated on the right sides of eqs. (2.6)-(2.14). Dashed lines denote scalar propagators. Solid lines denote fermion propagators with helicity along the direction of the arrow, and large dots between opposing arrows denote insertions of the fermion mass matrix M^{IJ} or its complex conjugate M^*_{IJ}, depending on whether the arrows are incoming or outgoing. Vector propagators are indicated by wavy lines, and ghost propagators by dotted lines. Each graph is also labelled by the type of propagators it contains, with S, F, V, g standing respectively for scalar, fermion, vector and ghost. Also, the presence of mass insertions in fermion lines is indicated by the overlines in the labels $\overline{FF}S$ and $\overline{FF}V$. The results for these Feynman diagrams (plus counterterms) are reported in sections 4, 5, and 6.

2.3 Two-loop integral functions needed for vacuum graphs

All of the effective potential two-loop integrals can be expressed in terms of linear combinations of functions introduced and studied by Ford, Jack and Jones in [7]. I will follow a notation similar but not identical to theirs: the functions $I(x, y, z)$, $J(x, y)$, and $J(x)$ used here are equal to the ϵ-independent parts of the functions $\hat{I}(x, y, z)$, $\hat{J}(x, y)$, and $J(x)$ used in ref. [7], up to obvious factors of $1/16\pi^2$. Explicitly, I choose to express results in terms of:

$$
\begin{aligned}
J(x) &= x(\overline{\ln}x - 1), && (2.17)\\
J(x,y) &= xy(\overline{\ln}x - 1)(\overline{\ln}y - 1), && (2.18)\\
I(x,y,z) &= \frac{1}{2}(x-y-z)\overline{\ln}y\overline{\ln}z + \frac{1}{2}(y-x-z)\overline{\ln}x\overline{\ln}z + \frac{1}{2}(z-x-y)\overline{\ln}x\overline{\ln}y \\
&\quad +2x\overline{\ln}x + 2y\overline{\ln}y + 2z\overline{\ln}z - \frac{5}{2}(x+y+z) - \frac{1}{2}\xi(x,y,z)\,. && (2.19)
\end{aligned}
$$

Here $\xi(x, y, z)$ was originally found in terms of Lobachevskiy's function or related integrals in ref. [7] using methods developed in [11, 12], but it can also be expressed [13, 14, 15] in terms of dilogarithms according to (for $x, y \leq z$):

$$
\begin{aligned}
\xi(x,y,z) &= R\Big\{2\ln[(z+x-y-R)/2z]\ln[(z+y-x-R)/2z] - \ln(x/z)\ln(y/z) \\
&\quad -2\mathrm{Li}_2[(z+x-y-R)/2z] - 2\mathrm{Li}_2[(z+y-x-R)/2z] + \pi^2/3\Big\} && (2.20)
\end{aligned}
$$

with

$$R = [x^2 + y^2 + z^2 - 2xy - 2xz - 2yz]^{1/2}. \qquad (2.21)$$

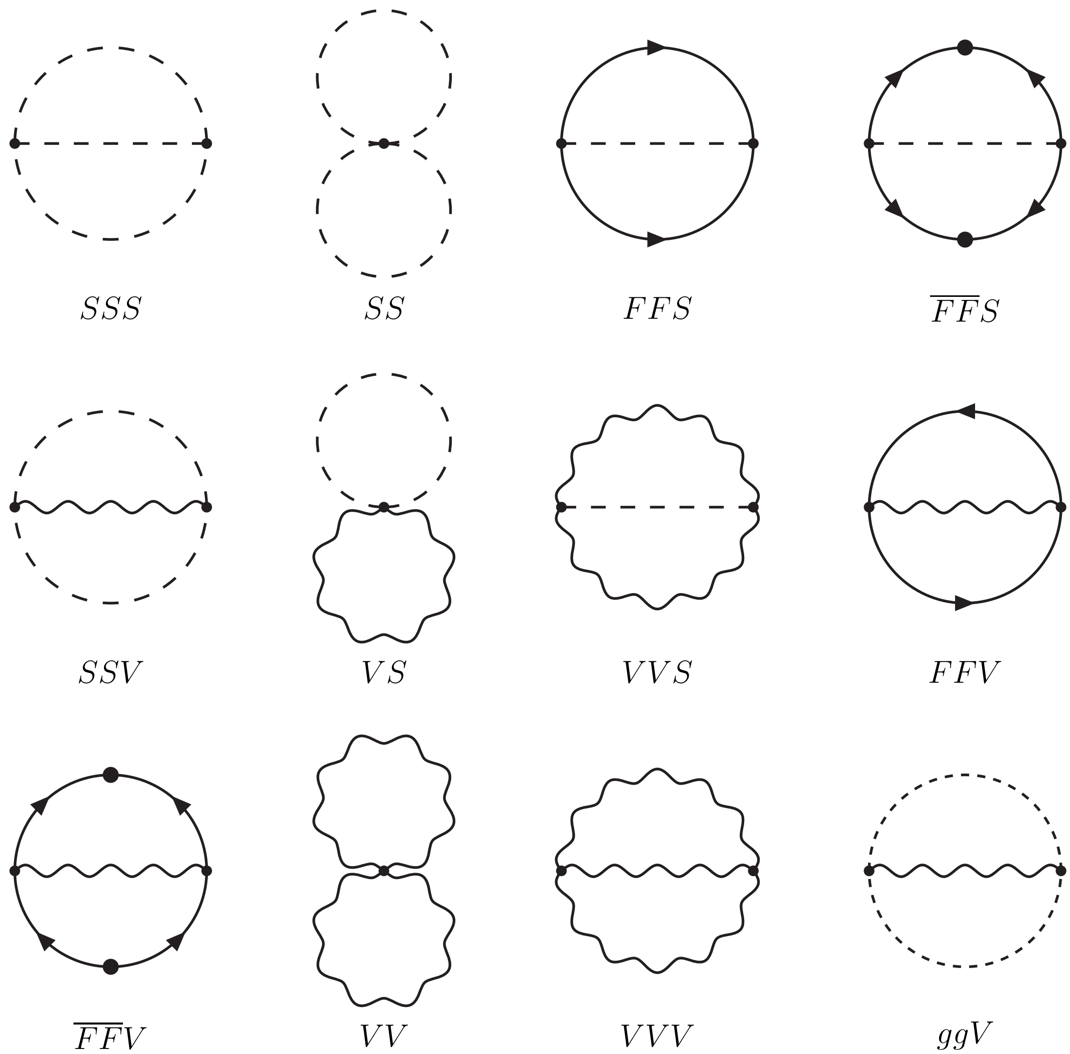

Figure 2: The one-particle-irreducible connected Feynman diagrams contributing to the two-loop effective potential. Dashed lines denote real scalars, solid lines denote Weyl fermions carrying helicity along the arrow direction, wavy lines are for vector bosons, and dotted lines are for ghosts. The large dots between opposing arrows on the fermion lines in the $\overline{FF}S$ and $\overline{FF}V$ diagrams denote mass insertions. The $\overline{FF}S$ diagram is accompanied by its complex conjugate (the same diagram with all arrows reversed).

The dilogarithm function is defined in the standard [16] way for any complex z:

$$\mathrm{Li}_2(z) = -\int_0^z \frac{\ln(1-t)}{t} dt. \tag{2.22}$$

To resolve branch cut ambiguities which could arise, consistently choose $\mathrm{Arg}(R) = 0$ or $\pi/2$ along with

$$-\pi < \mathrm{Im}[\ln(z)] \leq \pi \tag{2.23}$$

for all logarithms of negative or complex z, including the logarithm appearing in the integral definition of the dilogarithm. So, for example, when x is real and greater than 1,

$$\begin{aligned} \mathrm{Im}[\mathrm{Li}_2(x)] &= -i\pi \ln(x), & (2.24)\\ \mathrm{Im}[\mathrm{Li}_2(x \pm i\delta)] &= \pm i\pi \ln(x), & (2.25) \end{aligned}$$

for δ real and infinitesimal, while $\mathrm{Im}[\mathrm{Li}_2(x)] = 0$ if x is real and less than 1. The functions $\xi(x, y, z)$ and therefore $I(x, y, z)$ are invariant under interchange of any two of x, y, z.

It is useful to have expressions for these functions in the special cases of vanishing arguments. In addition to the trivial identities $J(0) = 0$, $J(x, 0) = J(0, x) = 0$ and $I(0, 0, 0) = 0$, one finds [15]:

$$\begin{aligned} I(0, x, y) &= (x-y)\left[\mathrm{Li}_2(y/x) - \ln(x/y)\overline{\ln}(x-y) + \frac{1}{2}(\overline{\ln}x)^2 - \frac{\pi^2}{6}\right] \\ &\quad -\frac{5}{2}(x+y) + 2x\overline{\ln}x + 2y\overline{\ln}y - x\overline{\ln}x\overline{\ln}y, & (2.26)\\ I(0, x, x) &= 2J(x) - 2x - \frac{1}{x}J(x, x) \;=\; -x(\overline{\ln}x)^2 + 4x\overline{\ln}x - 5x, & (2.27)\\ I(0, 0, x) &= -\frac{1}{2}x(\overline{\ln}x)^2 + 2x\overline{\ln}x - \frac{5}{2}x - \frac{\pi^2}{6}x. & (2.28) \end{aligned}$$

It is also sometimes useful to expand these functions for infinitesimal arguments:

$$\begin{aligned} I(\delta, x, y) &= I(0, x, y) + \delta\Big\{-(x+y)I(0, x, y) - 2J(x, y) + 3xJ(x) + 3yJ(y) - yJ(x) \\ &\quad -xJ(y) - (x+y)^2 + (x-y)[J(y) - J(x)]\overline{\ln}\delta\Big\}/(x-y)^2 + \ldots, & (2.29)\\ I(\delta, x, x) &= 2J(x) - 2x - \frac{1}{x}J(x, x) \\ &\quad +\delta\left\{4 + \frac{1}{2x^2}J(x, x) + \frac{3}{x}J(x) - [1 + \frac{J(x)}{x}]\overline{\ln}\delta\right\} + \ldots, & (2.30)\\ I(\delta_1, \delta_2, x) &= I(0, 0, x) + \frac{\delta_1}{x}\left[-x - I(0, 0, x) + 3J(x) - J(x)\overline{\ln}\delta_1\right] \\ &\quad +\frac{\delta_2}{x}\left[-x - I(0, 0, x) + 3J(x) - J(x)\overline{\ln}\delta_2\right] + \frac{\delta_1\delta_2}{x^2}\Big\{-2I(0, 0, x) \\ &\quad +4J(x) - 2x - [x + J(x)](\overline{\ln}\delta_1 + \overline{\ln}\delta_2) + x\overline{\ln}\delta_1\overline{\ln}\delta_2\Big\} + \ldots, & (2.31) \end{aligned}$$

where the ellipses stand for terms with more than one power of δ or either δ_1 or δ_2.

2.4 Conventions for softly-broken supersymmetric models

One of the main applications of the results of this paper is to models with softly broken supersymmetry, such as the Minimal Supersymmetric Standard Model (MSSM). Therefore I now list the relevant conventions to be used here for such models. In general, the superpotential is given in terms of the chiral superfields Φ_i by

$$W = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j, \tag{2.32}$$

and the soft supersymmetry-breaking part of the Lagrangian is

$$-\mathcal{L}_{\text{soft}} = (\frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + c^i\phi_i + \frac{1}{2}M\lambda_a\lambda_a + \text{c.c.}) + (m^2)_i^j\phi^{*i}\phi_j + \Lambda, \tag{2.33}$$

where the ϕ_i are the complex scalar field components of the Φ_i, and the λ_a are the two-component gaugino fermions with mass M. The parameter c^i can only appear if there is a gauge-singlet chiral superfield in the theory. Note the presence of a vacuum energy term Λ. This is required in order for the full effective potential to be RG invariant [17]-[20]. The two-loop beta function for Λ is obtained in section 7, and the beta functions for each of the other couplings at two-loop order are given in [21, 22, 9, 10], Flipping the heights on all indices of a coupling implies complex conjugation, so $Y_{ijk} = (Y^{ijk})^*$, $\mu_{ij} = (\mu^{ij})^*$, $a_{ijk} = (a^{ijk})^*$, etc. The representation matrices for the chiral superfields are denoted by $(T^a)_i^j$. They satisfy

$$[T^a, T^b] = if^{abc}T^c, \tag{2.34}$$

where f^{abc} are the totally antisymmetric structure constants of the gauge group G. Then

$$(T^aT^a)_i^j = C(i)\delta_i^j, \tag{2.35}$$

$$\text{Tr}[T^aT^b] = S(R)\delta^{ab}, \tag{2.36}$$

$$f^{acd}f^{bcd} = C_G\delta^{ab} \tag{2.37}$$

define the quadratic Casimir invariant $C(i)$ for each representation, the total Dynkin index $S(R)$ summed over all representations, and the Casimir invariant of the adjoint representation. The dimension of the adjoint representation is

$$d_G = \text{Tr}[C(i)]/S(R). \tag{2.38}$$

I use a normalization such that each fundamental representation of $SU(N)$ has $C(i) = (N^2-1)/2N$ and contributes $1/2$ to $S(R)$.

3 One-loop effective potential in the $\overline{\rm MS}$, $\overline{\rm DR}$, and $\overline{\rm DR}'$ schemes

In this section, I review the known answers for the one-loop effective potential. This will allow us to carefully distinguish the results in the $\overline{\rm MS}$, $\overline{\rm DR}$, and $\overline{\rm DR}'$ schemes.

In the $\overline{\rm MS}$ scheme and Landau gauge, one has

$$V^{(1)}_{\overline{\rm MS}} = V^{(1)}_S + V^{(1)}_F + V^{(1)}_V \tag{3.1}$$

where the different contributions arise from scalars, fermions, and vectors going around the loop in figure 1:

$$V^{(1)}_S = \frac{1}{4}\sum_i (m_i^2)^2(\overline{\ln} m_i^2 - 3/2), \tag{3.2}$$

$$V^{(1)}_F = -\frac{1}{2}\sum_I (m_I^2)^2(\overline{\ln} m_I^2 - 3/2), \tag{3.3}$$

$$V^{(1)}_V = \frac{3}{4}\sum_a (m_a^2)^2(\overline{\ln} m_a^2 - 5/6). \tag{3.4}$$

The appearance of 5/6 rather than 3/2 in $V^{(1)}_V$ is due to the fact that there are only $4-2\epsilon$, rather than 4, vector degrees of freedom in $\overline{\rm MS}$.

In the $\overline{\rm DR}$ scheme, one must include also the effects of the epsilon scalars. Now, it is tempting to assume that the epsilon scalars have the same field-dependent mass as their $4-2\epsilon$ vector counterparts. However, as pointed out in ref. [9], this is actually inconsistent except in models with exact supersymmetry, unless one sticks to only one fixed value of the renormalization scale Q, because the epsilon-scalar squared mass has a beta function which is not homogeneous. Therefore, in general one must allow the epsilon scalars to have squared-mass eigenvalues $\hat m_a^2$ which are distinct from the m_a^2 for the ordinary vectors. To be specific, consider the explicit form of the field-dependent squared-mass matrix for the ordinary $4-2\epsilon$ vector fields:

$$m^2_{ab} = 2g^2\phi^{*i}(T^aT^b)_i^j\phi_j. \tag{3.5}$$

This has eigenvalues m_a^2. For the epsilon-scalar squared-mass matrix, one has instead:

$$\hat m^2_{ab} = 2g^2\phi^{*i}(T^aT^b)_i^j\phi_j + \delta_{ab} m_\epsilon^2, \tag{3.6}$$

where m_ϵ^2 is an “evanescent” [23] parameter. This matrix requires an orthogonal diagonalization matrix $N^{(\epsilon)}$ which differs from $N^{(V)}$:

$$N^{(\epsilon)}_{ac}\hat m^2_{cd} N^{(\epsilon)}_{bd} = \delta_{ab}\hat m_a^2. \tag{3.7}$$

Unless supersymmetry is not explicitly broken, the eigenvalues $\hat{m}_a^2$ will in general differ from m_a^2, and the corresponding couplings of the squared-mass eigenstate epsilon scalars are different from the couplings of squared-mass eigenstate vectors, because $N^{(\epsilon)}$ differs from $N^{(V)}$.

In the $\overline{\rm DR}$ scheme, with epsilon scalars included, one now finds

$$V^{(1)}_{\overline{\rm DR}} = V^{(1)}_S + V^{(1)}_F + V^{(1)}_V + V^{(1)}_\epsilon \tag{3.8}$$

where $V^{(1)}_S$, $V^{(1)}_F$, $V^{(1)}_V$ are as before, and

$$V^{(1)}_\epsilon \;\;=\;\; -\frac{1}{2}\sum_a (\hat{m}_a^2)^2. \tag{3.9}$$

However, m_ϵ^2 is an additional parameter with no physically observable counterpart, and so its appearance in the effective potential is quite inconvenient. The functional form of the effective potential is also not directly physically observable, so there is no contradiction; m_ϵ^2 must cancel only from observable quantities. However, clearly one would like to avoid having to include a distinct epsilon-scalar mass in calculations in the first place. This problem was solved in the context of softly broken supersymmetric models in ref. [10] with the introduction of the $\overline{\rm DR}'$ scheme. The point is that one can remove the dependence of the full one-loop effective potential on m_ϵ^2 by redefining the ordinary scalar squared masses and the vacuum energy term appearing in the tree-level part eq. (2.33):

$$(m^2_{\overline{\rm DR}'})_i^j \;\;=\;\; (m^2_{\overline{\rm DR}})_i^j - \frac{1}{16\pi^2}\left[\delta_i^j 2g^2 C(i) m_\epsilon^2\right], \tag{3.10}$$

$$\Lambda_{\overline{\rm DR}'} \;\;=\;\; \Lambda_{\overline{\rm DR}} - \frac{1}{16\pi^2}\frac{(m_\epsilon^2)^2}{2}. \tag{3.11}$$

The result is the $\overline{\rm DR}'$ scheme, and the effective potential in this scheme is the one usually quoted in the literature (and often slightly incorrectly referred to as the $\overline{\rm DR}$ one):

$$V^{(1)}_{\overline{\rm DR}'} \;\;=\;\; \sum_n (-1)^{2s_n}(2s_n+1) h(m_n^2) = {\rm STr}\left[h(m_n^2)\right], \tag{3.12}$$

where

$$h(x) = \frac{x^2}{4}\left[\overline{\ln}(x) - 3/2\right], \tag{3.13}$$

and n runs over all real scalar, Weyl fermion, and vector degrees of freedom. The scalar squared masses occurring in eq. (3.12) are the ones following from the redefinition in eq. (3.10), and the vector squared masses are the eigenvalues of eq. (3.5). The $\overline{\rm DR}'$ effective potential is both manifestly supersymmetric when the soft terms vanish, and independent of the unphysical evanescent parameter m_ϵ^2 when the soft terms do not vanish. It is not hard to see that m_ϵ^2 is simultaneously banished from the equations which relate the physical pole masses to the Q-dependent running masses in the theory, so it has been successfully decoupled from all practical calculations. It would be quite clumsy to use the original $\overline{\rm DR}$ scheme in studies of realistic models like the MSSM, since in RG running and evaluation of the effective potential one would have to keep extra contributions from epsilon-scalar masses in order to avoid inconsistencies. Therefore the $\overline{\rm DR}'$ scheme is the preferred one.

After making this painful distinction, it must be admitted that the $\overline{\rm DR}'$ final result for the effective potential has exactly the same form that one would have obtained if one had naively set m_ϵ^2 equal to zero in the first place in the $\overline{\rm DR}$ scheme calculation. However, this naive procedure is technically inconsistent whenever RG running is involved [9] and does not work for other calculations involving epsilon scalars, so one should really distinguish between the two schemes as a matter of principle. The parameters appearing in the $\overline{\rm DR}'$ effective potential obey $\overline{\rm DR}'$ renormalization group equations, which differ from the $\overline{\rm DR}$ ones with m_ϵ^2 set equal to 0.

The procedure of going from the $\overline{\rm DR}$ scheme to the $\overline{\rm DR}'$ scheme is similar at two loops, and is described explicitly in section 6.

4 Two-loop effective potential in the $\overline{\rm MS}$ scheme

The two-loop effective potential in the $\overline{\rm MS}$ scheme for the general theory with masses and couplings described by eqs. (2.6)-(2.14) can be computed by the methods described in [7]. In fact, all of the hard work of evaluating the relevant Feynman loop integrals has already been accomplished there; no new types of integrals arise. Momentum integrals and vector indices each run over $4-2\epsilon$ dimensions. For each two-loop diagram, one must include counterterms for the various one-loop divergent subdiagrams. The result still includes single and double poles in ϵ, which are then simply removed by two-loop counterterms in modified minimal subtraction. The final result can be divided into parts corresponding to the various graphs of figure 2. Because the VV, VVV, and ggV graphs all involve the same field-dependent coupling g^{abc}, it is natural to combine their contributions into a pure gauge piece $V^{(2)}_{\rm gauge}$.

For the result, I find:

$$V^{(2)} \;=\; V^{(2)}_{SSS} + V^{(2)}_{SS} + V^{(2)}_{FFS} + V^{(2)}_{\overline{FF}S} + V^{(2)}_{SSV} + V^{(2)}_{VS} + V^{(2)}_{VVS}$$

$$+V^{(2)}_{FFV} + V^{(2)}_{\overline{FF}V} + V^{(2)}_{\rm gauge}, \tag{4.1}$$

where, in terms of the masses and couplings as specified in eqs. (2.6)-(2.14),

$$\begin{aligned}
V^{(2)}_{SSS} &= \frac{1}{12}(\lambda^{ijk})^2 f_{SSS}(m_i^2, m_j^2, m_k^2), & (4.2)\\
V^{(2)}_{SS} &= \frac{1}{8}\lambda^{iijj} f_{SS}(m_i^2, m_j^2), & (4.3)\\
V^{(2)}_{FFS} &= \frac{1}{2}|y^{IJk}|^2 f_{FFS}(m_I^2, m_J^2, m_k^2), & (4.4)\\
V^{(2)}_{\overline{FF}S} &= \frac{1}{4} y^{IJk} y^{I'J'k} M^*_{II'} M^*_{JJ'} f_{\overline{FF}S}(m_I^2, m_J^2, m_k^2) + \text{c.c.}, & (4.5)\\
V^{(2)}_{SSV} &= \frac{1}{4}(g^{aij})^2 f_{SSV}(m_i^2, m_j^2, m_a^2), & (4.6)\\
V^{(2)}_{VS} &= \frac{1}{4} g^{aaii} f_{VS}(m_a^2, m_i^2), & (4.7)\\
V^{(2)}_{VVS} &= \frac{1}{4}(g^{abi})^2 f_{VVS}(m_a^2, m_b^2, m_i^2), & (4.8)\\
V^{(2)}_{FFV} &= \frac{1}{2}|g^{aJ}_I|^2 f_{FFV}(m_I^2, m_J^2, m_a^2), & (4.9)\\
V^{(2)}_{\overline{FF}V} &= \frac{1}{2} g^{aJ}_I g^{aJ'}_{I'} M^{II'} M^*_{JJ'} f_{\overline{FF}V}(m_I^2, m_J^2, m_a^2), & (4.10)\\
V^{(2)}_{\rm gauge} &= \frac{1}{12}(g^{abc})^2 f_{\rm gauge}(m_a^2, m_b^2, m_c^2), & (4.11)
\end{aligned}$$

in which all indices on the right side are summed over. The loop-integral functions appearing here are given by:

$$\begin{aligned}
f_{SSS}(x,y,z) &= -I(x,y,z), & (4.12)\\
f_{SS}(x,y) &= J(x,y), & (4.13)\\
f_{FFS}(x,y,z) &= J(x,y) - J(x,z) - J(y,z) + (x+y-z)I(x,y,z), & (4.14)\\
f_{\overline{FF}S}(x,y,z) &= 2I(x,y,z), & (4.15)\\
f_{SSV}(x,y,z) &= \frac{1}{z}\Bigl\{(-x^2-y^2-z^2+2xy+2xz+2yz)I(x,y,z) + (x-y)^2 I(0,x,y) \\
&\quad +(y-x-z)J(x,z) + (x-y-z)J(y,z) + zJ(x,y)\Bigr\} \\
&\quad +2(x+y-z/3)J(z), & (4.16)\\
f_{VS}(x,y) &= 3J(x,y) + 2xJ(y), & (4.17)\\
f_{VVS}(x,y,z) &= \frac{1}{4xy}\Bigl\{(-x^2-y^2-z^2-10xy+2xz+2yz)I(x,y,z) \\
&\quad +(x-z)^2 I(0,x,z) + (y-z)^2 I(0,y,z) - z^2 I(0,0,z) \\
&\quad +(z-x-y)J(x,y) + yJ(x,z) + xJ(y,z)\Bigr\} \\
&\quad +\frac{1}{2}J(x) + \frac{1}{2}J(y) + 2J(z) - x - y - z, & (4.18)
\end{aligned}$$

$$
\begin{aligned}
f_{FFV}(x,y,z) &= \frac{1}{z}\Big\{(x^2+y^2-2z^2-2xy+xz+yz)I(x,y,z)-(x-y)^2I(0,x,y) \\
&\quad +(x-y-2z)J(x,z)+(y-x-2z)J(y,z)+2zJ(x,y)\Big\} \\
&\quad +2(-x-y+z/3)J(z)-2xJ(x)-2yJ(y)+(x+y)^2-z^2, \qquad (4.19)\\
f_{\overline{FF}V}(x,y,z) &= 6I(x,y,z)+2(x+y+z)-4J(x)-4J(y), \qquad (4.20)\\
f_{\rm gauge}(x,y,z) &= \frac{1}{4xyz}\Big\{(-x^4-8x^3y-8x^3z+32x^2yz+18y^2z^2)I(x,y,z) \\
&\quad +(y-z)^2(y^2+10yz+z^2)I(0,y,z)+x^2(2yz-x^2)I(0,0,x) \\
&\quad +(x^2-9y^2-9z^2+9xy+9xz+14yz)xJ(y,z) \\
&\quad +4x^3yz+48xy^2z^2+(22y+22z-16x/3)xyzJ(x)\Big\} \\
&\quad +(x\leftrightarrow y)+(x\leftrightarrow z). \qquad (4.21)
\end{aligned}
$$

Symmetry factors have been explicitly factored out of eqs. (4.2)-(4.11), but fermion-loop minus signs and other factors associated with the evaluation of the Feynman diagrams are contained in the definitions of the functions. The functions obey obvious symmetries: $f_{SSS}(x,y,z)$ and $f_{\rm gauge}(x,y,z)$ are invariant under interchange of any two of x,y,z, while $f_{SS}(x,y)$, $f_{FFS}(x,y,z)$, $f_{\overline{FF}S}(x,y,z)$, $f_{SSV}(x,y,z)$, $f_{VVS}(x,y,z)$, $f_{FFV}(x,y,z)$, and $f_{\overline{FF}V}(x,y,z)$ are each invariant under interchange of x,y.

The functions involving vector fields contain factors $1/x$, $1/y$, and $1/z$ which appear to be singular in the massless vector limit. This is due to the appearance in the Landau gauge of vector propagators

$$\frac{1}{i}\left(\frac{\eta^{\mu\nu}-p^\mu p^\nu/p^2}{p^2+m^2-i\epsilon}\right), \qquad (4.22)$$

which give rise to factors

$$\frac{1}{p^2(p^2+m^2)}=\frac{1}{m^2}\left(\frac{1}{p^2}-\frac{1}{p^2+m^2}\right) \qquad (4.23)$$

in the loop integrals. The massless limits are actually smooth, and arise often in practice. It is therefore useful to have explicit expressions for those massless limits that are not immediately obvious. Using eqs. (2.29)-(2.31), they are found to be:

$$
\begin{aligned}
f_{SSV}(x,y,0) &= (x+y)^2+3(x+y)I(x,y,0)+3J(x,y)-2xJ(x)-2yJ(y), \qquad (4.24)\\
f_{VVS}(x,0,z) &= -\frac{3x}{4}-\frac{z}{2}-\frac{3z}{4x}I(0,0,z)+(\frac{3z}{4x}-\frac{9}{4})I(0,x,z)+\frac{3}{4x}J(x,z) \\
&\quad +2J(z), \qquad (4.25)\\
f_{VVS}(0,0,z) &= -3I(0,0,z)+\frac{7}{2}J(z)-\frac{5z}{4}, \qquad (4.26)
\end{aligned}
$$

$$
\begin{aligned}
f_{FFV}(x,y,0) &= 0, & (4.27)\\
f_{\rm gauge}(x,y,0) &= \frac{1}{4xy}\Bigl\{(43x^2y+43xy^2-7x^3-7y^3)I(0,x,y) \\
&\quad +(2y+7x)x^2 I(0,0,x)+(2x+7y)y^2 I(0,0,y) \\
&\quad +(34xy-7x^2-7y^2)J(x,y)\Bigr\}+4x^2+4y^2+\frac{35}{2}xy \\
&\quad -\frac{19}{3}[xJ(x)+yJ(y)]+5[yJ(x)+xJ(y)], & (4.28)\\
f_{\rm gauge}(x,0,0) &= 13xI(0,0,x)-\frac{59}{6}xJ(x)+\frac{23}{4}x^2. & (4.29)
\end{aligned}
$$

All of the functions vanish (by dimensional analysis) whenever all arguments vanish.

It may also be of interest to see the individual contributions of the Feynman diagrams labelled VV, VVV, and ggV in figure 2, even though these can always be combined into $V^{(2)}_{\rm gauge}$. These contributions are listed in Appendix A.

The classic results of Ford, Jack and Jones for the Standard Model [7] are a particularly useful special case of those found in this section, with which I have checked agreement. In fact, each type of term that can occur in a general model in $\overline{\rm MS}$ does in fact arise in the Standard Model case; no new types of integrals arise, so that the results of eqs. (4.12)-(4.21) could be inferred from [7] by some forensic combinatorics. Their functions $A(x,y,z)$, $B(x,y,z)$, $C(x,y)$, $D(x,y,z)$, $E(x,y)$, $\Sigma(x,y)$, and $\Delta(x,y,z)$ are respectively equal to the functions $f_{SSV}(x,y,z)$, $-f_{VVS}(x,y,z)$, $f_{VS}(x,y)$, $-f_{FFV}(x,y,z)$, $\sqrt{xy}f_{\overline{FF}V}(x,y,z)$, $f_{VV}(x,y)$, and $-f_{VVV}(x,y,z)$ given in this section and in Appendix B. [Note that after the published errata of ref. [7], a few further minor typographical errors have been recently corrected in the eprint archive version.]

5 Two-loop effective potential in the $\overline{\rm DR}$ scheme

In this section, I report the results for the effective potential in the $\overline{\rm DR}$ scheme. These are obtained by keeping all 4 components of each vector field, but performing momentum integrations in $4-2\epsilon$ dimensions. The difference, compared to the results for $\overline{\rm MS}$, can be organized in terms of the extra epsilon scalars with multiplicity 2ϵ. Of course, the SSS, SS, FFS, and $\overline{FF}S$ diagrams in fig. 2 are unaffected by this procedure. Also, the SSV and ggV diagrams are unchanged in going from $\overline{\rm MS}$ to $\overline{\rm DR}$, because in those cases all the vector indices are contracted with a loop momentum. The VS, FFV and $\overline{FF}V$ diagrams yield new contributions, which we can call ϵS, $FF\epsilon$ and $\overline{FF}\epsilon$, when the vector line in each case is turned into an epsilon-scalar line. In the VVS diagram, a non-vanishing additional contribution arises only when both vectors are turned into epsilon scalars; call this contribution $\epsilon\epsilon S$. In the VV diagram, one or both of the vector lines can become an epsilon scalar, yielding contributions to be called ϵV and $\epsilon\epsilon$ respectively. Finally, in the VVV diagram, any two of the vector lines can be turned into epsilon-scalar lines, resulting in a contribution $\epsilon\epsilon V$.

As discussed in section 3, the couplings of epsilon scalars have exactly the form indicated for vectors in eqs. (2.12)-(2.14), except that when an epsilon scalar is involved, the rotation to the squared-mass eigenstate basis requires $N^{(\epsilon)}$ rather than $N^{(V)}$. This distinction is indicated by replacing the vector index $a, b, c, \ldots$ by an epsilon-scalar index $\hat{a}, \hat{b}, \hat{c}, \ldots$ on the couplings. For example [compare to eqs. (2.15)-(2.16)], the epsilon scalar-epsilon scalar-vector, epsilon scalar-vector-vector, and fermion-fermion-epsilon scalar couplings are

$$g^{\hat{a}bc} = g f^{efg} N^{(\epsilon)}_{ae} N^{(V)}_{bf} N^{(V)}_{cg}, \tag{5.1}$$

$$g^{\hat{a}\hat{b}c} = g f^{efg} N^{(\epsilon)}_{ae} N^{(\epsilon)}_{bf} N^{(V)}_{cg}, \tag{5.2}$$

$$g^{\hat{a}J}_{I} = g (T^b)^K_L N^{(F)*}_{JK} N^{(F)}_{IL} N^{(\epsilon)}_{ab}. \tag{5.3}$$

Then the result in the $\overline{\rm DR}$ scheme can be written

$$V^{(2)}_{\overline{\rm DR}} = V^{(2)}_{\overline{\rm MS}} + V^{(2)}_{\epsilon S} + V^{(2)}_{\epsilon\epsilon S} + V^{(2)}_{FF\epsilon} + V^{(2)}_{\overline{FF}\epsilon} + V^{(2)}_{\epsilon V} + V^{(2)}_{\epsilon\epsilon} + V^{(2)}_{\epsilon\epsilon V}, \tag{5.4}$$

where

$$V^{(2)}_{\epsilon S} = \frac{1}{4} g^{\hat{a}\hat{a}ii} f_{\epsilon S}(\hat{m}^2_a, m^2_i), \tag{5.5}$$

$$V^{(2)}_{\epsilon\epsilon S} = \frac{1}{4} (g^{\hat{a}\hat{b}i})^2 f_{\epsilon\epsilon S}(\hat{m}^2_a, \hat{m}^2_b, m^2_i), \tag{5.6}$$

$$V^{(2)}_{FF\epsilon} = \frac{1}{2} |g^{\hat{a}J}_{I}|^2 f_{FF\epsilon}(m^2_I, m^2_J, \hat{m}^2_a), \tag{5.7}$$

$$V^{(2)}_{\overline{FF}\epsilon} = \frac{1}{2} g^{\hat{a}J}_{I} g^{\hat{a}J'}_{I'} M^{II'} M^*_{JJ'} f_{\overline{FF}\epsilon}(m^2_I, m^2_J, \hat{m}^2_a), \tag{5.8}$$

$$V^{(2)}_{\epsilon V} = \frac{1}{2} (g^{\hat{a}bc})^2 f_{\epsilon V}(\hat{m}^2_a, m^2_b), \tag{5.9}$$

$$V^{(2)}_{\epsilon\epsilon} = \frac{1}{4} (g^{\hat{a}\hat{b}c})^2 f_{\epsilon\epsilon}(\hat{m}^2_a, \hat{m}^2_b), \tag{5.10}$$

$$V^{(2)}_{\epsilon\epsilon V} = \frac{1}{4} (g^{\hat{a}\hat{b}c})^2 f_{\epsilon\epsilon V}(\hat{m}^2_a, \hat{m}^2_b, m^2_c), \tag{5.11}$$

with the loop functions given by:

$$f_{\epsilon S}(x, y) = -2xJ(y), \tag{5.12}$$

$$f_{\epsilon\epsilon S}(x, y, z) = -2J(z) + x + y + z, \tag{5.13}$$

$$f_{FF\epsilon}(x, y, z) = 2xJ(x) + 2yJ(y) - (x+y)^2 + z^2, \tag{5.14}$$

$$f_{\overline{FF}\epsilon}(x,y,z) = 4J(x)+4J(y)-2x-2y-2z, \tag{5.15}$$

$$f_{\epsilon V}(x,y) = -4xy-6xJ(y), \tag{5.16}$$

$$f_{\epsilon\epsilon}(x,y) = 4xy, \tag{5.17}$$

$$f_{\epsilon\epsilon V}(x,y,z) = -x^2-y^2+z^2-6xy-xz-yz+(6x+6y-2z)J(z). \tag{5.18}$$

This completes the result for the two-loop effective potential in the $\overline{\rm DR}$ scheme.

6 Two-loop effective potential in the $\overline{\rm DR}'$ scheme

As explained in the Introduction and in section 3, it is convenient in models of softly broken supersymmetry to go to the $\overline{\rm DR}'$ scheme. This scheme is defined so that m_ϵ^2 (the difference between the squared masses of epsilon scalars and their vector counterparts) does not appear in the beta functions of other couplings, or in the effective potential, or in the equations relating pole masses to running masses. Starting from the $\overline{\rm DR}$ results of the previous section, I find that this is done at two-loop order by the following parameter redefinition of soft terms appearing in eq. (2.33):

$$\begin{aligned}(m^2_{\overline{\rm DR}'})_i^j &= (m^2_{\overline{\rm DR}})_i^j - \frac{1}{16\pi^2}\left[\delta_i^j 2g^2C(i)m_\epsilon^2\right] + \frac{1}{(16\pi^2)^2}\Big\{Y^{ikl}Y_{jkl}g^2[C(k)-\frac{1}{2}C(i)]m_\epsilon^2 \\ &\qquad +\delta_i^j g^4C(i)\left[2S(R)+4C(i)-6C_G\right]m_\epsilon^2\Big\},\end{aligned} \tag{6.1}$$

$$c^i_{\overline{\rm DR}'} = c^i_{\overline{\rm DR}} + \frac{1}{(16\pi^2)^2}\left[Y^{ijk}\mu_{jk}g^2C(j)m_\epsilon^2\right], \tag{6.2}$$

$$\begin{aligned}\Lambda_{\overline{\rm DR}'} &= \Lambda_{\overline{\rm DR}} - \frac{1}{16\pi^2}\frac{(m_\epsilon^2)^2}{2} + \frac{1}{(16\pi^2)^2}\Big\{\frac{g^2}{2}d_G[S(R)-C_G](m_\epsilon^2)^2 \\ &\qquad +g^2d_GC_G|M|^2m_\epsilon^2 + g^2\mu^{ij}\mu_{ij}C(i)m_\epsilon^2\Big\}.\end{aligned} \tag{6.3}$$

If there is more than one simple or $U(1)$ group, then each of the correction terms should be summed over subgroups. Following these redefinitions, the result for the full two-loop effective potential turns out to have the same functional form as if one naively took the $\overline{\rm DR}$ result and set m_ϵ^2 to 0, removing the distinction between $N^{(\epsilon)}$ and $N^{(V)}$, between hatted and unhatted vector squared-mass eigenstate indices on the couplings, and between $\hat{m}_a^2$ and m_a^2. It is therefore convenient to define functions which combine the effects of the $4-2\epsilon$ vectors and the epsilon scalars:

$$F_{VS}(x,y) = f_{VS}(x,y)+f_{\epsilon S}(x,y), \tag{6.4}$$

$$F_{VVS}(x,y,z) = f_{VVS}(x,y,z)+f_{\epsilon\epsilon S}(x,y,z), \tag{6.5}$$

$$F_{FFV}(x,y,z) = f_{FFV}(x,y,z)+f_{FF\epsilon}(x,y,z), \tag{6.6}$$

$$
\begin{aligned}
F_{\overline{FF}V}(x,y,z) &= f_{\overline{FF}V}(x,y,z) + f_{\overline{FF}\epsilon}(x,y,z), &(6.7)\\
F_{\rm gauge}(x,y,z) &= f_{\rm gauge}(x,y,z) + f_{\epsilon\epsilon V}(x,y,z) + f_{\epsilon\epsilon V}(z,x,y) + f_{\epsilon\epsilon V}(y,z,x)\\
&\quad + f_{\epsilon V}(x,y) + f_{\epsilon V}(y,x) + f_{\epsilon V}(x,z) + f_{\epsilon V}(z,x) + f_{\epsilon V}(y,z) + f_{\epsilon V}(z,y)\\
&\quad + f_{\epsilon\epsilon}(x,y) + f_{\epsilon\epsilon}(x,z) + f_{\epsilon\epsilon}(y,z). &(6.8)
\end{aligned}
$$

Note that I use F's rather than f's to distinguish the $\overline{\rm DR}'$ functions from the corresponding $\overline{\rm MS}$ functions.

Therefore, the $\overline{\rm DR}'$ two-loop effective potential is given by:

$$
\begin{aligned}
V^{(2)} &= V^{(2)}_{SSS} + V^{(2)}_{SS} + V^{(2)}_{FFS} + V^{(2)}_{\overline{FF}S} + V^{(2)}_{SSV} + V^{(2)}_{VS} + V^{(2)}_{VVS}\\
&\quad + V^{(2)}_{FFV} + V^{(2)}_{\overline{FF}V} + V^{(2)}_{\rm gauge}, &(6.9)
\end{aligned}
$$

where now

$$
\begin{aligned}
V^{(2)}_{SSS} &= \frac{1}{12}(\lambda^{ijk})^2 f_{SSS}(m_i^2, m_j^2, m_k^2), &(6.10)\\
V^{(2)}_{SS} &= \frac{1}{8}\lambda^{iijj} f_{SS}(m_i^2, m_j^2), &(6.11)\\
V^{(2)}_{FFS} &= \frac{1}{2}|y^{IJk}|^2 f_{FFS}(m_I^2, m_J^2, m_k^2), &(6.12)\\
V^{(2)}_{\overline{FF}S} &= \frac{1}{4}y^{IJk}y^{I'J'k}M^*_{II'}M^*_{JJ'}f_{\overline{FF}S}(m_I^2, m_J^2, m_k^2) + \text{c.c.}, &(6.13)\\
V^{(2)}_{SSV} &= \frac{1}{4}(g^{aij})^2 f_{SSV}(m_i^2, m_j^2, m_a^2), &(6.14)\\
V^{(2)}_{VS} &= \frac{1}{4}g^{aaii} F_{VS}(m_a^2, m_i^2), &(6.15)\\
V^{(2)}_{VVS} &= \frac{1}{4}(g^{abi})^2 F_{VVS}(m_a^2, m_b^2, m_i^2), &(6.16)\\
V^{(2)}_{FFV} &= \frac{1}{2}|g^{aJ}_I|^2 F_{FFV}(m_I^2, m_J^2, m_a^2), &(6.17)\\
V^{(2)}_{\overline{FF}V} &= \frac{1}{2}g^{aJ}_I g^{aJ'}_{I'} M^{II'} M^*_{JJ'} F_{\overline{FF}V}(m_I^2, m_J^2, m_a^2), &(6.18)\\
V^{(2)}_{\rm gauge} &= \frac{1}{12}(g^{abc})^2 F_{\rm gauge}(m_a^2, m_b^2, m_c^2). &(6.19)
\end{aligned}
$$

Here $f_{SSS}(x,y,z)$, $f_{SS}(x,y)$, $f_{FFS}(x,y,z)$, $f_{\overline{FF}S}(x,y,z)$, and $f_{SSV}(x,y,z)$ are given by exactly the same functions as in $\overline{\rm MS}$, eqs. (4.12)-(4.16). The new functions are given by:

$$
\begin{aligned}
F_{VS}(x,y) &= 3J(x,y), &(6.20)\\
F_{VVS}(x,y,z) &= \frac{1}{4xy}\Big\{(-x^2 - y^2 - z^2 - 10xy + 2xz + 2yz)I(x,y,z)\\
&\quad + (x-z)^2 I(0,x,z) + (y-z)^2 I(0,y,z) - z^2 I(0,0,z)\\
&\quad + (z-x-y)J(x,y) + yJ(x,z) + xJ(y,z)\Big\}
\end{aligned}
$$

$$+\frac{1}{2}J(x)+\frac{1}{2}J(y), \tag{6.21}$$

$$\begin{aligned} F_{FFV}(x,y,z) &= \frac{1}{z}\Big\{(x^2+y^2-2z^2-2xy+xz+yz)I(x,y,z)-(x-y)^2I(0,x,y)\\ &\quad +(x-y-2z)J(x,z)+(y-x-2z)J(y,z)+2zJ(x,y)\Big\}\\ &\quad +2(-x-y+z/3)J(z), \end{aligned} \tag{6.22}$$

$$F_{\overline{FF}V}(x,y,z) = 6I(x,y,z), \tag{6.23}$$

$$\begin{aligned} F_{\rm gauge}(x,y,z) &= \frac{1}{4xyz}\Big\{(-x^4-8x^3y-8x^3z+32x^2yz+18y^2z^2)I(x,y,z)\\ &\quad +(y-z)^2(y^2+10yz+z^2)I(0,y,z)+x^2(2yz-x^2)I(0,0,x)\\ &\quad +(x^2-9y^2-9z^2+9xy+9xz+14yz)xJ(y,z)\\ &\quad +(22y+22z-40x/3)xyzJ(x)\Big\}\\ &\quad +(x\leftrightarrow y)+(x\leftrightarrow z). \end{aligned} \tag{6.24}$$

Despite the appearance of x, y, z in the denominators, these functions again all have smooth limits for $x, y, z \rightarrow 0$. The non-trivial ones are

$$F_{VVS}(x,0,z) = \frac{x}{4}+\frac{z}{2}-\frac{3z}{4x}I(0,0,z)+(\frac{3z}{4x}-\frac{9}{4})I(0,x,z)+\frac{3}{4x}J(x,z), \tag{6.25}$$

$$F_{VVS}(0,0,z) = -3I(0,0,z)+\frac{3}{2}J(z)-\frac{z}{4}, \tag{6.26}$$

$$F_{FFV}(x,y,0) = -(x+y)^2+2xJ(x)+2yJ(y), \tag{6.27}$$

$$\begin{aligned} F_{\rm gauge}(x,y,0) &= \frac{1}{4xy}\Big\{(43x^2y+43xy^2-7x^3-7y^3)I(0,x,y)\\ &\quad +(2y+7x)x^2I(0,0,x)+(2x+7y)y^2I(0,0,y)\\ &\quad +(34xy-7x^2-7y^2)J(x,y)\Big\}+3x^2+3y^2+\frac{11}{2}xy\\ &\quad -\frac{25}{3}[xJ(x)+yJ(y)]+5[yJ(x)+xJ(y)], \end{aligned} \tag{6.28}$$

$$F_{\rm gauge}(0,0,x) = 13xI(0,0,x)-\frac{71}{6}xJ(x)+\frac{19}{4}x^2. \tag{6.29}$$

Also, it may be of interest to see the contributions from individual graphs to $F_{\rm gauge}(x,y,z)$. Those are listed in Appendix A.

This completes the result for the two-loop effective potential in the $\overline{\rm DR}'$ scheme. These are appropriate for use in any softly broken supersymmetric model, including the MSSM. Partial results for the MSSM corresponding to the leading contributions proportional to $\alpha_S y_t^2$ and y_t^4 have been given in refs. [30] and [15]. Several illustrative examples and consistency checks are done in section 8.

7 Renormalization group invariance of the two-loop effective potential in softly broken supersymmetry

In general, the condition for RG invariance of the effective potential is

$$Q\frac{dV}{dQ} = \left(Q\frac{\partial}{\partial Q} + \sum_I \beta_{\lambda_I}\frac{\partial}{\partial \lambda_I} - \sum_i \gamma_i^{(S)}\phi_i\frac{\partial}{\partial\phi_i}\right) V = 0. \tag{7.1}$$

Here, λ_I are all of the running parameters of the model with beta functions β_{λ_I}, and $\gamma_i^{(S)}$ are the anomalous dimensions of the scalar fields ϕ_i. At one- and two-loop order, this means

$$Q\frac{\partial}{\partial Q}V^{(1)} + \left[\sum_I \beta^{(1)}_{\lambda_I}\frac{\partial}{\partial \lambda_I} - \sum_i \gamma_i^{(S,1)}\phi_i\frac{\partial}{\partial\phi_i}\right] V^{(0)} \;=\; 0, \tag{7.2}$$

$$\begin{aligned} Q\frac{\partial}{\partial Q}V^{(2)} &+ \left[\sum_I \beta^{(1)}_{\lambda_I}\frac{\partial}{\partial \lambda_I} - \sum_i \gamma_i^{(S,1)}\phi_i\frac{\partial}{\partial\phi_i}\right] V^{(1)} \\ &+ \left[\sum_I \beta^{(2)}_{\lambda_I}\frac{\partial}{\partial \lambda_I} - \sum_i \gamma_i^{(S,2)}\phi_i\frac{\partial}{\partial\phi_i}\right] V^{(0)} \;=\; 0. \end{aligned} \tag{7.3}$$

In softly broken supersymmetry, I find that the anomalous dimension matrix for scalar fields in the Landau gauge and in either $\overline{\rm DR}$ or $\overline{\rm DR}'$ is

$$\gamma_i^{(S)j} = \frac{1}{16\pi^2}\gamma_i^{(S,1)j} + \frac{1}{(16\pi^2)^2}\gamma_i^{(S,2)j}; \tag{7.4}$$

$$\gamma_i^{(S,1)j} = \frac{1}{2}Y_{ikl}Y^{jkl} - \delta_i^j g^2 C(i), \tag{7.5}$$

$$\begin{aligned}\gamma_i^{(S,2)j} =& -\frac{1}{2}Y_{imn}Y^{nkl}Y_{klr}Y^{mrj} + Y_{ikl}Y^{jkl}g^2[2C(k) - C(i)] \\ &+\delta_i^j g^4 C(i)[S(R) + 2C(i) - \frac{9}{4}C_G]. \end{aligned}\tag{7.6}$$

This can be obtained starting from the general results in the $\overline{\rm MS}$ scheme in ref. [24], and then applying the coupling constant redefinitions needed to transform from the $\overline{\rm MS}$ to the $\overline{\rm DR}$ or $\overline{\rm DR}'$ scheme [25]. The eigenvalues of this matrix constrained to the subspace of the classical scalar background fields give the anomalous dimensions appearing in eqs. (7.2) and (7.3). It should be noted that because of gauge-fixing, the Landau gauge scalar field anomalous dimension matrix $\gamma_i^{(S)j}$ relevant for the effective potential is *not* the same as the more widely-known, gauge-invariant, anomalous dimension matrix of the chiral superfields. For comparison, the latter is [26]

$$\gamma_i^{j} = \frac{1}{16\pi^2}\gamma_i^{(1)j} + \frac{1}{(16\pi^2)^2}\gamma_i^{(2)j}; \tag{7.7}$$

$$\gamma_i^{(1)j} = \frac{1}{2}Y_{ikl}Y^{jkl} - 2\delta_i^j g^2 C(i), \tag{7.8}$$

$$\gamma_i^{(2)j} = -\frac{1}{2} Y_{imn} Y^{nkl} Y_{klr} Y^{mrj} + Y_{ikl} Y^{jkl} g^2 [2C(k) - C(i)] \\ +\delta_i^j g^4 C(i) [2S(R) + 4C(i) - 6C_G]. \tag{7.9}$$

In order for the effective potential to satisfy eq. (7.1) in a model with explicit supersymmetry breaking, it is necessary to include a running vacuum energy term Λ, as in eq. (2.33). Now using the results of section 6, one can derive the $\overline{\rm DR}'$ beta function for Λ up to two loops in a general softly-broken supersymmetric model as specified in subsection 2.4, by looking at the ϕ_i-independent piece of eqs. (7.2)-(7.3). I find

$$\beta_\Lambda = \frac{1}{16\pi^2} \beta_\Lambda^{(1)} + \frac{1}{(16\pi^2)^2} \beta_\Lambda^{(2)} \tag{7.10}$$

$$\beta_\Lambda^{(1)} = (m^2)_i^j (m^2)_j^i + 2(m^2)_i^j \mu^{ik} \mu_{kj} + b^{ij} b_{ij} - d_G |M|^4, \tag{7.11}$$

$$\begin{aligned} \beta_\Lambda^{(2)} = {} & g^2 d_G |M|^4 [4S(R) - 8C_G] + 8g^2 |M|^2 \mu^{ij} \mu_{ij} C(i) + 8g^2 (m^2)_i^j \mu^{ik} \mu_{kj} C(i) \\ & +4g^2 (m^2)_i^j (m^2)_j^i C(i) + 4g^2 b^{ij} b_{ij} C(i) - 4g^2 M \mu^{ij} b_{ij} C(i) - 4g^2 M^* \mu_{ij} b^{ij} C(i) \\ & -Y^{ijk} Y_{ijl} \left[(m^2)_k^m (m^2)_m^l + (m^2)_k^m \mu_{mn} \mu^{nl} + \mu_{km} \mu^{mn} (m^2)_n^l + \mu_{km} (m^2)_n^m \mu^{nl} + b_{km} b^{ml} \right] \\ & -a^{ijk} a_{ijl} \left[(m^2)_k^l + \mu_{km} \mu^{ml} \right] - 2Y^{ijk} Y_{ilm} (m^2)_j^l \mu^{mn} \mu_{nk} \\ & -Y^{ijk} a_{ijl} \mu_{km} b^{ml} - Y_{ijk} a^{ijl} \mu^{km} b_{ml}. \end{aligned} \tag{7.12}$$

where d_G is the dimension of the adjoint representation of the group. If the gauge group contains more than one simple or $U(1)$ component, then terms involving the gaugino mass M or g^2 should be summed over subgroups in eqs. (7.5)-(7.6), (7.8)-(7.9), and (7.11)-(7.12), Special cases of these general results will be used in the next section.

8 Examples and consistency checks

In this section, I study some examples chosen as consistency checks and useful points of reference for the results given above. The examples are all based on supersymmetry with or without soft breaking, so the $\overline{\rm DR}'$ scheme is used. One type of consistency check follows from the requirement that the two-loop effective potential satisfies RG invariance in conjunction with the known two-loop beta functions [21, 22, 9, 10], and the scalar anomalous dimensions and β_Λ found in the previous section. The derivatives of the loop functions are listed in Appendix B. Another type of check relies on the fact that the effective potential for a supersymmetric theory in a supersymmetric vacuum must vanish. These consistency checks rely on non-trivial cancellations between different two-loop functions, which are made manifest by writing them in terms of the basis functions $I(x, y, z)$, $J(x, y)$, and $J(x)$, using eqs. (4.12)-(4.16) and (6.20)-(6.29).

8.1 The Wess-Zumino Model

Consider the Wess-Zumino model [27] with a single chiral supermultiplet Φ containing a Weyl fermion ψ and a complex scalar $\phi + (R + iI)/\sqrt{2}$, where ϕ is the classical background, and R, I are real scalar quantum fluctuations. The superpotential is given by

$$W = \frac{\mu}{2}\Phi^2 + \frac{y}{6}\Phi^3, \tag{8.1}$$

where μ and y are mass and coupling parameters, taken to be real without loss of generality. The fields R, I, ψ are mass eigenstates, with

$$m_R^2 = \mu^2 + 3y\mu\phi + 3y^2\phi^2/2, \tag{8.2}$$

$$m_I^2 = \mu^2 + y\mu\phi + y^2\phi^2/2, \tag{8.3}$$

$$m_\psi = \mu + y\phi. \tag{8.4}$$

The tree-level scalar potential is

$$V^{(0)} = \mu^2\phi^2 + y\mu\phi^3 + y^2\phi^4/4, \tag{8.5}$$

and the one-loop contribution to the effective potential is given in terms of the function $h(x)$ in eq. (3.13) by

$$V^{(1)} = h(m_R^2) + h(m_I^2) - 2h(m_\psi^2). \tag{8.6}$$

The non-zero scalar quartic and cubic couplings are:

$$\lambda^{RRRR} = \lambda^{IIII} = 3y^2/2, \tag{8.7}$$

$$\lambda^{RRII} = y^2/2, \tag{8.8}$$

$$\lambda^{RRR} = 3y(\mu + y\phi)/\sqrt{2}, \tag{8.9}$$

$$\lambda^{RII} = y(\mu + y\phi)/\sqrt{2}, \tag{8.10}$$

and the Yukawa interactions are

$$y^{\psi\psi R} = y/\sqrt{2}, \tag{8.11}$$

$$y^{\psi\psi I} = iy/\sqrt{2}. \tag{8.12}$$

It follows that the contributions to the two-loop effective potential are:

$$V_{SSS}^{(2)} = \frac{y^2}{8}(\mu + y\phi)^2\left[3f_{SSS}(m_R^2, m_R^2, m_R^2) + f_{SSS}(m_R^2, m_I^2, m_I^2)\right], \tag{8.13}$$

$$
\begin{aligned}
V^{(2)}_{SS} &= \frac{y^2}{16}\left[3f_{SS}(m_R^2,m_R^2)+3f_{SS}(m_I^2,m_I^2)+2f_{SS}(m_R^2,m_I^2)\right], &(8.14)\\
V^{(2)}_{FFS} &= \frac{y^2}{4}\left[f_{FFS}(m_\psi^2,m_\psi^2,m_R^2)+f_{FFS}(m_\psi^2,m_\psi^2,m_I^2)\right], &(8.15)\\
V^{(2)}_{\overline{FF}S} &= \frac{y^2}{4}m_\psi^2\left[f_{\overline{FF}S}(m_\psi^2,m_\psi^2,m_R^2)-f_{\overline{FF}S}(m_\psi^2,m_\psi^2,m_I^2)\right]. &(8.16)
\end{aligned}
$$

Now one may check RG invariance of the effective potential. At one-loop order, one finds from eq. (8.6) that

$$
Q\frac{\partial}{\partial Q}V^{(1)} = -y^2\mu^2\phi^2 - y^3\mu\phi^3 - y^4\phi^4/4. \tag{8.17}
$$

The one-loop scalar anomalous dimension and beta functions are

$$
\begin{aligned}
\gamma_\phi^{(S,1)} &= y^2/2, &(8.18)\\
\beta_\mu^{(1)} &= y^2\mu, &(8.19)\\
\beta_y^{(1)} &= 3y^3/2. &(8.20)
\end{aligned}
$$

Therefore, from eq. (8.5):

$$
\begin{aligned}
\sum_I \beta_{\lambda_I}^{(1)}\frac{\partial}{\partial\lambda_I}V^{(0)} &= 2y^2\mu^2\phi^2 + 5y^3\mu\phi^3/2 + 3y^4\phi^4/4, &(8.21)\\
-\gamma_\phi^{(S,1)}\phi\frac{\partial}{\partial\phi}V^{(0)} &= -y^2\mu^2\phi^2 - 3y^3\mu\phi^3/2 - y^4\phi^4/2, &(8.22)
\end{aligned}
$$

where λ_I runs over y, μ, so that eq. (7.2) is indeed satisfied. At two-loop order, one finds from eqs. (8.13)-(8.16) and (B.5)-(B.8) that

$$
Q\frac{\partial}{\partial Q}V^{(2)} + \left[\sum_I \beta_{\lambda_I}^{(1)}\frac{\partial}{\partial\lambda_I} - \gamma_\phi^{(S,1)}\phi\frac{\partial}{\partial\phi}\right]V^{(1)} = y^4\mu^2\phi^2 + y^5\mu\phi^3 + y^6\phi^4/4. \tag{8.23}
$$

From the two-loop RG scalar anomalous dimension and beta functions:

$$
\begin{aligned}
\gamma_\phi^{(S,2)} &= -y^4/2, &(8.24)\\
\beta_\mu^{(2)} &= -y^4\mu, &(8.25)\\
\beta_y^{(2)} &= -3y^5/2, &(8.26)
\end{aligned}
$$

one also finds:

$$
\begin{aligned}
\sum_I \beta_{\lambda_I}^{(2)}\frac{\partial}{\partial\lambda_I}V^{(0)} &= -2y^4\mu^2\phi^2 - 5y^5\mu\phi^3/2 - 3y^6\phi^4/4, &(8.27)\\
-\gamma_\phi^{(S,2)}\phi\frac{\partial}{\partial\phi}V^{(0)} &= y^4\mu^2\phi^2 + 3y^5\mu\phi^3/2 + y^6\phi^4/2. &(8.28)
\end{aligned}
$$

The results of eqs. (8.23), (8.27) and (8.28) combine to verify eq. (7.3).

In the special case of $\phi = 0$, supersymmetry is not broken, and the effective potential should vanish. At one-loop order, eq. (8.6) then vanishes trivially. At two-loop order,

$$V^{(2)} = \frac{y^2}{2}\left[\mu^2 f_{SSS}(\mu^2,\mu^2,\mu^2) + f_{SS}(\mu^2,\mu^2) + f_{FFS}(\mu^2,\mu^2,\mu^2)\right], \tag{8.29}$$

which equals 0 by virtue of eqs. (4.12)-(4.14).

8.2 Supersymmetric QED in supersymmetric vacua

Let us now consider a supersymmetric $U(1)$ gauge theory with coupling constant g and a pair of chiral superfields with charges ± 1.

First take the case that the chiral superfields do not have a mass term before symmetry breaking, and the two scalar fields have the same classical background value ϕ. Then the gauge symmetry is broken, but supersymmetry remains unbroken since ϕ parameterizes a flat direction. The vector boson, two Weyl fermions, and a real scalar field each obtain a mass

$$x = 4g^2\phi^2. \tag{8.30}$$

Together with a massless (in Landau gauge) real scalar Nambu-Goldstone boson, these form a massive vector supermultiplet. In addition, there are two massless real scalars and one massless Weyl fermion forming a chiral supermultiplet. The $\overline{\mathrm{DR}}'$ one-loop effective potential vanishes because of these mass degeneracies. The two-loop effective potential contributions in the $\overline{\mathrm{DR}}'$ scheme are:

$$V^{(2)}_{SSS} = g^2 x\left[\frac{1}{2} f_{SSS}(0,0,x) + f_{SSS}(0,x,x)\right], \tag{8.31}$$

$$V^{(2)}_{FFS} = g^2\left[f_{FFS}(0,x,0) + f_{FFS}(0,x,x) + 2 f_{FFS}(x,x,0)\right], \tag{8.32}$$

$$V^{(2)}_{SSV} = \frac{g^2}{2}\left[f_{SSV}(0,0,x) + f_{SSV}(0,x,x)\right], \tag{8.33}$$

$$V^{(2)}_{VS} = \frac{g^2}{2} F_{VS}(x,x), \tag{8.34}$$

$$V^{(2)}_{VVS} = g^2 x F_{VVS}(x,x,0), \tag{8.35}$$

$$V^{(2)}_{FFV} = g^2 F_{FFV}(0,x,x), \tag{8.36}$$

with the other contributions vanishing. One can now check by plugging in the results of section 6 that the sum of eqs. (8.31)-(8.36) yields 0, as required for a supersymmetric vacuum. This constitutes a non-trivial identity involving cancellations between different two-loop functions which become apparent after writing them in terms of the functions $I(x,y,z)$, $J(x,y)$ and $J(x)$.

Another check which relies on a different set of cancellations is obtained if we take $\phi = 0$ in the above model, but now include a superpotential mass term μ. In that case, the vector gauge boson and the gaugino are massless, and the real scalar fields and the chiral fermions all have squared mass μ^2. Then one obtains for the contributions to the two-loop effective potential in the $\overline{\rm DR}'$ scheme:

$$V^{(2)}_{SS} = g^2 f_{SS}(\mu^2,\mu^2), \tag{8.37}$$
$$V^{(2)}_{FFS} = 4g^2 f_{FFS}(0,\mu^2,\mu^2), \tag{8.38}$$
$$V^{(2)}_{SSV} = g^2 f_{SSV}(\mu^2,\mu^2,0), \tag{8.39}$$
$$V^{(2)}_{FFV} = g^2 F_{FFV}(\mu^2,\mu^2,0), \tag{8.40}$$
$$V^{(2)}_{\overline{FF}V} = -g^2\mu^2 F_{\overline{FF}V}(\mu^2,\mu^2,0), \tag{8.41}$$

with all other contributions vanishing. Again one finds from the results of section 6 that the sum of eqs. (8.37)-(8.41) yields 0, as required for a supersymmetric vacuum.

8.3 Supersymmetric $SU(N_c)$ gauge theory with one flavor in supersymmetric vacua

A richer set of checks is found in non-abelian supersymmetric models. As an example, consider supersymmetric $SU(N_c)$ gauge theory with one flavor of chiral superfields Q_i and $\overline{Q}^i$ in the fundamental and anti-fundamental representations, respectively. Here $i = 1,\ldots,N_c$ is a color index. Consider evaluation of the effective potential for the classical background:

$$\langle Q_i\rangle = \langle \overline{Q}^i\rangle = \delta_{i1}\phi. \tag{8.42}$$

These VEVs break the gauge symmetry according to $SU(N_c) \to SU(N_c-1)$, but ϕ parameterizes a flat direction and supersymmetry is unbroken. Therefore the effective potential must vanish at each order in perturbation theory for any value of ϕ. My aim is to show this explicitly.

The particle content for non-zero ϕ consists of $2N_c-1$ massive vector supermultiplets with their associated massless (in Landau gauge) real scalar Nambu-Goldstone modes, $N_c^2 - 2N_c$ massless vector multiplets associated with the unbroken gauge symmetry, and one massless singlet chiral supermultiplet. The non-zero squared-mass eigenvalues are

$$x = g^2\phi^2, \tag{8.43}$$

$$y = \frac{2(N_c-1)}{N_c} g^2\phi^2, \tag{8.44}$$

and the multiplicities of the mass eigenstates are shown in Table 1. Because of the mass

Table 1: Multiplicities of mass eigenstates in the model of section 8.3.

particle type	$m^2 = 0$	$m^2 = x$	$m^2 = y$
real scalars	$2N_c + 1$	$2N_c - 2$	1
Weyl fermions	$N_c^2 - 2N_c + 1$	$4N_c - 4$	2
vectors	$N_c^2 - 2N_c$	$2N_c - 2$	1

degeneracies indicated in Table 1, the one-loop contribution to the effective potential vanishes as required.

At two-loop order, I find the contributions in the $\overline{\rm DR}'$ scheme to be

$$\begin{aligned}
V^{(2)}_{SSS} &= g^4\phi^2\Bigl[\frac{N_c-1}{4}f_{SSS}(0,0,x) + \frac{(N_c-1)^2}{2N_c^2}f_{SSS}(0,0,y) + \frac{N_c-1}{2}f_{SSS}(0,x,x) \\
&\quad + \frac{(N_c-1)^2}{N_c^2}f_{SSS}(0,y,y) + \frac{(N_c-2)^2(N_c-1)}{4N_c^2}f_{SSS}(0,x,y)\Bigr], && (8.45)\\
V^{(2)}_{SS} &= 0, && (8.46)\\
V^{(2)}_{FFS} &= g^2\Bigl[\frac{2N_c^2-3N_c-1}{2}\left\{f_{FFS}(0,x,0)+f_{FFS}(0,x,x)\right\} + \frac{3(N_c-1)}{2}f_{FFS}(x,x,0) \\
&\quad + \frac{N_c-1}{2N_c}\left\{f_{FFS}(0,y,0)+f_{FFS}(0,y,y)\right\} + \frac{N_c-1}{N_c}f_{FFS}(y,y,0) \\
&\quad + \frac{N_c^2-N_c+2}{2N_c}\left\{f_{FFS}(x,y,0)+f_{FFS}(x,y,x)\right\} + \frac{N_c-1}{2}f_{FFS}(x,x,y)\Bigr], && (8.47)\\
V^{(2)}_{\overline{FF}S} &= g^4\phi^2\Bigl[\frac{N_c-1}{2}\left\{f_{\overline{FF}S}(x,x,y)-f_{\overline{FF}S}(x,x,0)\right\} \\
&\quad + \frac{2(N_c-1)}{N_c}\left\{f_{\overline{FF}S}(x,y,x)-f_{\overline{FF}S}(x,y,0)\right\}\Bigr], && (8.48)\\
V^{(2)}_{SSV} &= g^2\Bigl[\frac{N_c-1}{2}f_{SSV}(0,0,x) + \frac{1}{4}f_{SSV}(0,0,y) + \frac{N_c-1}{4N_c}f_{SSV}(0,y,y) \\
&\quad + \frac{N_c-1}{4}f_{SSV}(0,x,x) + \frac{N_c(N_c-2)}{4}f_{SSV}(x,x,0) + \frac{1}{4N_c}f_{SSV}(x,x,y) \\
&\quad + \frac{N_c-1}{4}f_{SSV}(x,y,x)\Bigr], && (8.49)\\
V^{(2)}_{VS} &= g^2\Bigl[\frac{N_c-1}{2}F_{VS}(x,x) + \frac{N_c-1}{4}F_{VS}(x,y) + \frac{1}{2N_c}F_{VS}(y,x)
\end{aligned}$$

$$
\begin{aligned}
& + \frac{N_c - 1}{4N_c} F_{VS}(y,y)\Big], && (8.50) \\
V^{(2)}_{VVS} &= g^4\phi^2 \Big[\frac{N_c(N_c-2)}{2} F_{VVS}(0,x,0) + \frac{N_c - 1}{2} F_{VVS}(x,x,0) \\
& + \frac{(N_c-2)^2}{2N_c} F_{VVS}(x,y,0) + \frac{(N_c-1)^2}{N_c^2} F_{VVS}(y,y,0)\Big], && (8.51) \\
V^{(2)}_{FFV} &= g^2 \Big[\frac{2N_c^2 - 3N_c - 1}{2} F_{FFV}(0,x,x) + N_c(N_c-2) F_{FFV}(x,x,0) \\
& + \frac{N_c - 1}{2N_c} F_{FFV}(0,y,y) + \frac{N_c^2+1}{2N_c} F_{FFV}(x,x,y) + \frac{3N_c - 1}{2} F_{FFV}(x,y,x)\Big], && (8.52) \\
V^{(2)}_{\overline{FF}V} &= -g^4\phi^2 \Big[N_c(N_c-2) F_{\overline{FF}V}(x,x,0) + F_{\overline{FF}V}(x,x,y) \\
& + 2(N_c - 1) F_{\overline{FF}V}(x,y,x)\Big], && (8.53) \\
V^{(2)}_{\rm gauge} &= g^2 \Big[\frac{N_c(N_c-2)}{4} F_{\rm gauge}(0,x,x) + \frac{N_c}{4} F_{\rm gauge}(x,x,y)\Big]. && (8.54)
\end{aligned}
$$

After some algebra, using eqs. (4.12)-(4.16) and (6.20)-(6.29), one finds that the sum of these contributions indeed vanishes, as required by unbroken supersymmetry in the flat direction parameterized by ϕ.

8.4 Softly-broken supersymmetric QED

Consider the case of supersymmetric QED with a coupling g and two chiral superfields with charges ± 1, as in subsection 8.2. However, now we introduce supersymmetry-breaking effects in the form of a gaugino mass M, and non-holomorphic soft supersymmetry-breaking scalar squared masses m_+^2 and m_-^2 for the scalar fields of charge $+1$, -1 respectively. Instead of equal VEVs, the scalar fields of charge $+1$, -1 are taken to have classical background values ϕ, 0 respectively. Then the four real scalar mass eigenstates obtain squared masses x_1, x_1, x_2, x_3 where

$$
\begin{aligned}
x_1 &= m_-^2 - g^2\phi^2, && (8.55) \\
x_2 &= m_+^2 + g^2\phi^2, && (8.56) \\
x_3 &= m_+^2 + 3g^2\phi^2, && (8.57)
\end{aligned}
$$

and the three fermion mass eigenstates obtain squared masses $0, y_1, y_2$, with

$$
\begin{aligned}
y_1 &= [M^2 + 4g^2\phi^2 - M\sqrt{M^2 + 8g^2\phi^2}]/2, && (8.58) \\
y_2 &= [M^2 + 4g^2\phi^2 + M\sqrt{M^2 + 8g^2\phi^2}]/2, && (8.59)
\end{aligned}
$$

while the vector boson obtains a mass

$$z = 2g^2\phi^2. \tag{8.60}$$

Because supersymmetry is explicitly broken, RG invariance requires that a vacuum-energy Λ is included among the soft supersymmetry breaking terms. The tree-level potential is then:

$$V^{(0)} = \Lambda + m_+^2\phi^2 + \frac{g^2}{2}\phi^4. \tag{8.61}$$

From eq. (3.13), the $\overline{\rm DR}'$ one-loop effective potential contribution is:

$$V^{(1)} = 2h(x_1) + h(x_2) + h(x_3) - 2h(y_1) - 2h(y_2) + 3h(z). \tag{8.62}$$

In that scheme, by following the procedures described in sections 2.1 and 6, I find the following contributions to the two-loop effective potential:

$$\begin{aligned}
V^{(2)}_{SSS} &= g^4\phi^2\Big[f_{SSS}(x_1,x_1,x_3) + \frac{1}{2}f_{SSS}(x_2,x_2,x_3) + \frac{3}{2}f_{SSS}(x_3,x_3,x_3)\Big], && (8.63)\\
V^{(2)}_{SS} &= g^2\Big[f_{SS}(x_1,x_1) - \frac{1}{2}f_{SS}(x_1,x_2) - \frac{1}{2}f_{SS}(x_1,x_3) + \frac{3}{8}f_{SS}(x_2,x_2) \\
&\quad + \frac{1}{4}f_{SS}(x_2,x_3) + \frac{3}{8}f_{SS}(x_3,x_3)\Big], && (8.64)\\
V^{(2)}_{FFS} &= \frac{g^2}{M^2+8g^2\phi^2}\Big[2(y_1+z)f_{FFS}(0,y_1,x_1) + 2(y_2+z)f_{FFS}(0,y_2,x_1) \\
&\quad + 2z\{f_{FFS}(y_1,y_1,x_2) + f_{FFS}(y_1,y_1,x_3) + f_{FFS}(y_2,y_2,x_2) + f_{FFS}(y_2,y_2,x_3)\} \\
&\quad + M^2\{f_{FFS}(y_1,y_2,x_2) + f_{FFS}(y_1,y_2,x_3)\}\Big], && (8.65)\\
V^{(2)}_{\overline{FF}S} &= \frac{2g^4\phi^2}{M^2+8g^2\phi^2}\Big[2y_1\{f_{\overline{FF}S}(y_1,y_1,x_3) - f_{\overline{FF}S}(y_1,y_1,x_2)\} \\
&\quad + 2y_2\{f_{\overline{FF}S}(y_2,y_2,x_3) - f_{\overline{FF}S}(y_2,y_2,x_2)\} \\
&\quad + M^2\{f_{\overline{FF}S}(y_1,y_2,x_2) - f_{\overline{FF}S}(y_1,y_2,x_3)\}\Big], && (8.66)\\
V^{(2)}_{SSV} &= \frac{g^2}{2}\Big[f_{SSV}(x_1,x_1,z) + f_{SSV}(x_2,x_3,z)\Big], && (8.67)\\
V^{(2)}_{VS} &= g^2\Big[F_{VS}(z,x_1) + \frac{1}{2}F_{VS}(z,x_2) + \frac{1}{2}F_{VS}(z,x_3)\Big], && (8.68)\\
V^{(2)}_{VVS} &= g^2 z F_{VVS}(z,z,x_3), && (8.69)\\
V^{(2)}_{FFV} &= \frac{g^2}{2(M^2+8g^2\phi^2)}\Big[(M^2+8g^2\phi^2)F_{FFV}(0,0,z) + y_2F_{FFV}(y_1,y_1,z) \\
&\quad + y_1F_{FFV}(y_2,y_2,z) + 2zF_{FFV}(y_1,y_2,z)\Big], && (8.70)\\
V^{(2)}_{\overline{FF}V} &= \frac{2g^6\phi^4}{M^2+8g^2\phi^2}\Big[F_{\overline{FF}V}(y_1,y_1,z) + F_{\overline{FF}V}(y_2,y_2,z) - 2F_{\overline{FF}V}(y_1,y_2,z)\Big], && (8.71)\\
V^{(2)}_{\rm gauge} &= 0. && (8.72)
\end{aligned}$$

We can now test the RG invariance of the effective potential. The one-loop scalar anomalous dimension and beta functions in the $\overline{\rm DR}'$ scheme are:

$$
\begin{aligned}
\gamma_\phi^{(S,1)} &= -g^2, && (8.73)\\
\beta_g^{(1)} &= 2g^3, && (8.74)\\
\beta_M^{(1)} &= 4g^2 M, && (8.75)\\
\beta_{m_+^2}^{(1)} &= -8g^2M^2 + 2g^2(m_+^2 - m_-^2), && (8.76)\\
\beta_{m_-^2}^{(1)} &= -8g^2M^2 + 2g^2(m_-^2 - m_+^2), && (8.77)\\
\beta_\Lambda^{(1)} &= (m_+^2)^2 + (m_-^2)^2 - M^4. && (8.78)
\end{aligned}
$$

From eq. (8.62) one therefore finds that

$$
\begin{aligned}
Q\frac{\partial}{\partial Q}V^{(1)} &= M^4 - (m_+^2)^2 - (m_-^2)^2 + 8g^2\phi^2M^2 + 2g^2\phi^2 m_-^2\\
&\quad -4g^2\phi^2 m_+^2 - 4g^4\phi^4, && (8.79)
\end{aligned}
$$

and, from eqs. (8.73)-(8.78),

$$
\begin{aligned}
\sum_I \beta_{\lambda_I}^{(1)}\frac{\partial}{\partial\lambda_I}V^{(0)} &= -M^4 + (m_+^2)^2 + (m_-^2)^2 - 8g^2\phi^2M^2 - 2g^2\phi^2m_-^2\\
&\quad +2g^2\phi^2m_+^2 + 2g^4\phi^4, && (8.80)\\
-\gamma_\phi^{(S,1)}\phi\frac{\partial}{\partial\phi}V^{(0)} &= 2g^2\phi^2m_+^2 + 2g^4\phi^4, && (8.81)
\end{aligned}
$$

so that eq. (7.2) is satisfied. At two loop order, one has

$$
\begin{aligned}
\gamma_\phi^{(S,2)} &= 4g^4, && (8.82)\\
\beta_g^{(2)} &= 8g^5, && (8.83)\\
\beta_M^{(2)} &= 32g^4M, && (8.84)\\
\beta_{m_+^2}^{(2)} &= 96g^4M^2 + 16g^4m_+^2, && (8.85)\\
\beta_{m_-^2}^{(2)} &= 96g^4M^2 + 16g^4m_-^2, && (8.86)\\
\beta_\Lambda^{(2)} &= 4g^2(m_+^2)^2 + 4g^2(m_-^2)^2 + 8g^2M^4, && (8.87)
\end{aligned}
$$

so that

$$
\begin{aligned}
\sum_I \beta_{\lambda_I}^{(2)}\frac{\partial}{\partial\lambda_I}V^{(0)} &= 8g^2M^4 + 4g^2(m_+^2)^2 + 4g^2(m_-^2)^2 + 96g^4\phi^2M^2\\
&\quad +16g^4\phi^2m_+^2 + 8g^6\phi^4, && (8.88)\\
-\gamma_\phi^{(S,2)}\phi\frac{\partial}{\partial\phi}V^{(0)} &= -8g^4\phi^2m_+^2 - 8g^6\phi^4. && (8.89)
\end{aligned}
$$

One also finds from eqs. (8.63)-(8.72) and the results of section 6:

$$Q\frac{\partial}{\partial Q}V^{(2)} + \left[\sum_I \beta_{\lambda_I}^{(1)}\frac{\partial}{\partial \lambda_I} - \gamma_\phi^{(S,1)}\phi\frac{\partial}{\partial\phi}\right]V^{(1)} = -8g^2M^4 - 4g^2(m_+^2)^2 - 4g^2(m_-^2)^2 -96g^4M^2\phi^2 - 8g^4\phi^2 m_+^2. \tag{8.90}$$

Together, eqs. (8.88)-(8.90) verify eq. (7.3).

9 Outlook

In this paper, I have presented the results for the two-loop effective potential of a general renormalizable field theory in the Landau gauge, in each of the $\overline{\rm MS}$, $\overline{\rm DR}$, and $\overline{\rm DR}'$ renormalization schemes. These results should be useful in connecting specific models of electroweak symmetry breaking to future data in a precise way.

It is not unlikely that the correct model for physics near the TeV scale is based on some version of softly-broken supersymmetry, either the MSSM or some moderate extension of it. Previous calculations of the effective potential in the MSSM have used the one-loop result [28] and partial two-loop approximations with leading corrections proportional to $\alpha_S y_t^4$ and y_t^4 [29]-[33]. However, there is still some RG scale-dependence in these results, compared to estimates of our eventual ability to measure properties of the Higgs sector at future colliders. Use of the full two-loop $\overline{\rm DR}'$ effective potential should further reduce the scale dependence. RG improvement methods [17]-[20], [34]-[38] should enable an accurate determination of the vacuum of the MSSM and its extensions. I plan to report on the application of the results of the present paper to the MSSM soon [39].

Appendix A: Individual diagram contributions to the functions $f_{\rm gauge}$ and $F_{\rm gauge}$

The three Feynman diagrams labelled VV, VVV, and ggV in figure 2 all involve the field-dependent coupling g^{abc}, and combine to yield $V^{(2)}_{\rm gauge}$. In the $\overline{\rm MS}$ scheme, the individual diagram contributions to the function $f_{\rm gauge}(x,y,z)$ are given in an obvious notation by

$$f_{\rm gauge}(x,y,z) = f_{VVV}(x,y,z) + f_{VV}(x,y) + f_{VV}(x,z) + f_{VV}(y,z) + f_{ggV}(x) + f_{ggV}(y) + f_{ggV}(z), \tag{A.1}$$

where

$$f_{VVV}(x,y,z) = \frac{1}{4xyz}\Big\{(-x^4 - 8x^3y - 8x^3z + 32x^2yz + 18y^2z^2)I(x,y,z)$$

$$
\begin{aligned}
& +(y-z)^2(y^2+10yz+z^2)I(0,y,z)-x^4I(0,0,x) \\
& +(x^2-9y^2-9z^2+9xy+9xz-13yz)xJ(y,z) \\
& +4x^3yz+\frac{129}{4}xy^2z^2-\left(\frac{20x}{3}+\frac{y}{2}+\frac{z}{2}\right)xyzJ(x)\Big\} \\
& +(x\leftrightarrow y)+(x\leftrightarrow z), \qquad \text{(A.2)}
\end{aligned}
$$

$$
f_{VV}(x,y) = \frac{27}{4}J(x,y)+\frac{45x}{8}J(y)+\frac{45y}{8}J(x)+\frac{63xy}{16}, \tag{A.3}
$$

$$
f_{ggV}(x) = \frac{x}{2}I(0,0,x)+\frac{x}{3}J(x). \tag{A.4}
$$

Similarly, in the $\overline{\rm DR}'$ scheme,

$$
\begin{aligned}
F_{\rm gauge}(x,y,z) &= F_{VVV}(x,y,z)+F_{VV}(x,y)+F_{VV}(x,z)+F_{VV}(y,z) \\
&\quad +f_{ggV}(x)+f_{ggV}(y)+f_{ggV}(z), \qquad \text{(A.5)}
\end{aligned}
$$

where

$$
F_{VVV}(x,y,z) = f_{VVV}(x,y,z)+f_{\epsilon\epsilon V}(x,y,z)+f_{\epsilon\epsilon V}(z,x,y)+f_{\epsilon\epsilon V}(y,z,x), \tag{A.6}
$$

$$
F_{VV}(x,y) = f_{VV}(x,y)+f_{\epsilon V}(x,y)+f_{\epsilon V}(y,x)+f_{\epsilon\epsilon}(x,y). \tag{A.7}
$$

with $f_{ggV}(x)$ given as before. Explicitly,

$$
\begin{aligned}
F_{VVV}(x,y,z) &= \frac{1}{4xyz}\Big\{(-x^4-8x^3y-8x^3z+32x^2yz+18y^2z^2)I(x,y,z) \\
&\quad +(y-z)^2(y^2+10yz+z^2)I(0,y,z)-x^4I(0,0,x) \\
&\quad +(x^2-9y^2-9z^2+9xy+9xz-13yz)xJ(y,z) \\
&\quad +\frac{xy^2z^2}{4}+(-\frac{44x}{3}+\frac{47y}{2}+\frac{47z}{2})xyzJ(x)\Big\} \\
&\quad +(x\leftrightarrow y)+(x\leftrightarrow z), \qquad \text{(A.8)}
\end{aligned}
$$

$$
F_{VV}(x,y) = \frac{27}{4}J(x,y)-\frac{3x}{8}J(y)-\frac{3y}{8}J(x)-\frac{xy}{16}. \tag{A.9}
$$

The results for vanishing arguments are easily obtained from eqs. (2.29)-(2.31).

Appendix B: Renormalization-group-scale derivatives

It is often useful to have expressions for the derivatives of the two-loop effective potential functions with respect to the renormalization scale Q, for example to check RG invariance. The derivative of the one-loop effective potential function $h(x)$ defined in eq. (3.13) is

$$
Q\frac{\partial}{\partial Q}h(x) = -x^2/2. \tag{B.1}
$$

The derivatives of the two-loop functions can all be found from those of the basis functions:

$$Q\frac{\partial}{\partial Q}J(x) = -2x, \tag{B.2}$$

$$Q\frac{\partial}{\partial Q}J(x,y) = 2xy(2-\overline{\ln}x-\overline{\ln}y), \tag{B.3}$$

$$Q\frac{\partial}{\partial Q}I(x,y,z) = 2(x\overline{\ln}x+y\overline{\ln}y+z\overline{\ln}z)-4(x+y+z). \tag{B.4}$$

For the derivatives of the $\overline{\rm MS}$ two-loop functions, one finds:

$$Q\frac{\partial}{\partial Q}f_{SSS}(x,y,z) = -2(x\overline{\ln}x+y\overline{\ln}y+z\overline{\ln}z)+4(x+y+z), \tag{B.5}$$

$$Q\frac{\partial}{\partial Q}f_{SS}(x,y) = 2xy(2-\overline{\ln}x-\overline{\ln}y), \tag{B.6}$$

$$\begin{aligned} Q\frac{\partial}{\partial Q}f_{FFS}(x,y,z) &= 4(z^2-x^2-y^2-xy-xz-yz)+2x^2\overline{\ln}x+2y^2\overline{\ln}y \\ &\quad -2z^2\overline{\ln}z+4(x+y)z\overline{\ln}z, \end{aligned} \tag{B.7}$$

$$Q\frac{\partial}{\partial Q}f_{\overline{FF}S}(x,y,z) = 4(x\overline{\ln}x+y\overline{\ln}y+z\overline{\ln}z)-8(x+y+z), \tag{B.8}$$

$$\begin{aligned} Q\frac{\partial}{\partial Q}f_{SSV}(x,y,z) &= -8x^2-8y^2+\frac{16}{3}z^2-12(xy+xz+yz)+6x^2\overline{\ln}x+6y^2\overline{\ln}y \\ &\quad -2z^2\overline{\ln}z+6(x+y)z\overline{\ln}z, \end{aligned} \tag{B.9}$$

$$Q\frac{\partial}{\partial Q}f_{VS}(x,y) = xy(8-6\overline{\ln}x-6\overline{\ln}y), \tag{B.10}$$

$$Q\frac{\partial}{\partial Q}f_{VVS}(x,y,z) = 9(x+y)+5z-\frac{9}{2}(x\overline{\ln}x+y\overline{\ln}y)-6z\overline{\ln}z, \tag{B.11}$$

$$Q\frac{\partial}{\partial Q}f_{FFV}(x,y,z) = \frac{20}{3}z^2+(6x+6y-4z)z\overline{\ln}z, \tag{B.12}$$

$$Q\frac{\partial}{\partial Q}f_{\overline{FF}V}(x,y,z) = 12(x\overline{\ln}x+y\overline{\ln}y+z\overline{\ln}z)-16(x+y)-24z, \tag{B.13}$$

$$\begin{aligned} Q\frac{\partial}{\partial Q}f_{\rm gauge}(x,y,z) &= -\frac{97}{3}(x^2+y^2+z^2)-72(xy+xz+yz) \\ &\quad +\frac{9}{2}[xy(\overline{\ln}x+\overline{\ln}y)+xz(\overline{\ln}x+\overline{\ln}z)+yz(\overline{\ln}y+\overline{\ln}z)] \\ &\quad +26(x^2\overline{\ln}x+y^2\overline{\ln}y+z^2\overline{\ln}z). \end{aligned} \tag{B.14}$$

For the functions used with epsilon scalars in the $\overline{\rm DR}$ scheme, one has:

$$Q\frac{\partial}{\partial Q}f_{\epsilon S}(x,y) = 4xy, \tag{B.15}$$

$$Q\frac{\partial}{\partial Q}f_{\epsilon\epsilon S}(x,y,z) = 4z, \tag{B.16}$$

$$Q\frac{\partial}{\partial Q}f_{FF\epsilon}(x,y,z) = -4x^2-4y^2, \tag{B.17}$$

$$Q\frac{\partial}{\partial Q}f_{\overline{FF}\epsilon}(x,y,z) = -8x-8y, \tag{B.18}$$

$$Q\frac{\partial}{\partial Q}f_{\epsilon V}(x,y) = 12xy, \tag{B.19}$$

$$Q\frac{\partial}{\partial Q}f_{\epsilon\epsilon}(x,y) = 0, \tag{B.20}$$

$$Q\frac{\partial}{\partial Q}f_{\epsilon\epsilon V}(x,y,z) = -12xz-12yz+4z^2. \tag{B.21}$$

Finally, the functions used in the $\overline{\rm DR}'$ scheme (besides those found in $\overline{\rm MS}$) satisfy:

$$Q\frac{\partial}{\partial Q}F_{VS}(x,y) = 6xy(2-\overline{\ln}x-\overline{\ln}y), \tag{B.22}$$

$$Q\frac{\partial}{\partial Q}F_{VVS}(x,y,z) = 9(x+y+z)-\frac{9}{2}(x\overline{\ln}x+y\overline{\ln}y)-6z\overline{\ln}z, \tag{B.23}$$

$$Q\frac{\partial}{\partial Q}F_{FFV}(x,y,z) = -4x^2-4y^2+\frac{20}{3}z^2+(6x+6y-4z)z\overline{\ln}z, \tag{B.24}$$

$$Q\frac{\partial}{\partial Q}F_{\overline{FF}V}(x,y,z) = 12(x\overline{\ln}x+y\overline{\ln}y+z\overline{\ln}z)-24(x+y+z), \tag{B.25}$$

$$\begin{aligned} Q\frac{\partial}{\partial Q}F_{\rm gauge}(x,y,z) &= -\frac{85}{3}(x^2+y^2+z^2)-72(xy+xz+yz) \\ &\quad +\frac{9}{2}[xy(\overline{\ln}x+\overline{\ln}y)+xz(\overline{\ln}x+\overline{\ln}z)+yz(\overline{\ln}y+\overline{\ln}z)] \\ &\quad +26(x^2\overline{\ln}x+y^2\overline{\ln}y+z^2\overline{\ln}z). \end{aligned} \tag{B.26}$$

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